英

令和元年度 秋季募集
(令和 2 年 4 月入学)
東北大学大学院機械系 4 専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

令和元年 8 月 27 日(火)
Tuesday, August 27, 2019　　9:30 − 11:00

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Answer all problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学 A　MATHEMATICS A

1. Solve the following problems.

   (1) Evaluate the indefinite integral

$$\int \frac{2x}{x^4 + x^2 + 1}\, dx.$$

   (2) Evaluate the definite integral

$$\int_1^3 \frac{x^2}{x^2 - 4x + 5}\, dx.$$

   (3) Evaluate the infinite integral

$$\int_0^\infty \frac{x^2}{(x^2 + 2x + 2)^2}\, dx.$$

数学 A　MATHEMATICS A

2. The 3×3 matrix A is given by

$$A = \begin{pmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 2 \end{pmatrix}.$$

Solve the following problems.

(1) Find the inverse matrix of A.

(2) Find the eigenvalues and eigenvectors of A.

(3) Find coefficients a, b, and c such that $A^3 + aA^2 + bA + cI = O$, where I and O are the identity matrix and the zero matrix, respectively.

(4) Obtain $A^4 - 6A^3 + 7A^2 - 8A + 2I$.

数学 A　MATHEMATICS A

3. In the Cartesian coordinate system (x, y, z), the vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \frac{x}{\sqrt{x^2+y^2}}\boldsymbol{i} + \frac{y}{\sqrt{x^2+y^2}}\boldsymbol{j} + \frac{z}{\sqrt{x^2+y^2}}\boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors in the x, y, and z directions, respectively. Let S be the surface of the region V given by

$$V = \{\ (x, y, z)\ |\ a^2 \leq x^2+y^2 \leq b^2,\ \ 0 \leq z \leq c\ \},$$

where a, b, and c are real numbers satisfying $0 < a < b$ and $c > 0$. Solve the following problems.

(1) Obtain $\nabla \cdot \boldsymbol{A}$.

(2) Obtain $\nabla \times \boldsymbol{A}$.

Introduce the cylindrical coordinate system (r, θ, z) as shown in Fig. 1 and let $\boldsymbol{e}_r$, $\boldsymbol{e}_\theta$, and $\boldsymbol{e}_z$ be the fundamental vectors in the r, θ, and z directions, respectively. Solve the following problems.

(3) Represent $\boldsymbol{i}$ and $\boldsymbol{j}$ in the cylindrical coordinate system.

(4) Represent $\boldsymbol{A}$ in the cylindrical coordinate system.

(5) Evaluate the integral

$$\int_S \boldsymbol{A} \cdot \boldsymbol{n}\, dS,$$

where $\boldsymbol{n}$ is the outward unit normal vector of S.

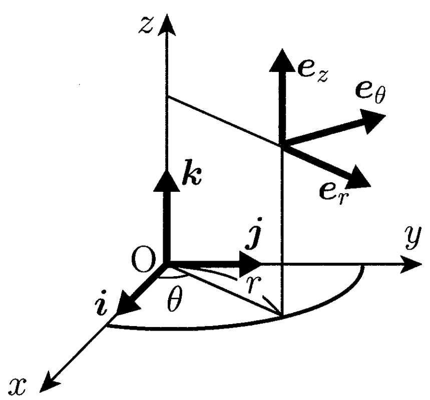

Fig. 1

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(令和 2 年 4 月入学)
東北大学大学院機械系 4 専攻入学試験

試験問題冊子

数学 B　MATHEMATICS B

令和元年 8 月 27 日(火)
Tuesday, August 27, 2019　　13:30 — 15:00

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Answer all problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\dfrac{dy}{dx} = (y - 4x + 1)^2$

(2) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 5y = \sin x$

(3) $x^2\dfrac{d^2y}{dx^2} - 3x\dfrac{dy}{dx} + 4y = \log x \qquad (x > 0)$

数学B　MATHEMATICS B

2. The Fourier transform $F(\omega)$ of a function $f(x)$ and its inverse transform are defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx\,,$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\, e^{i\omega x}\, d\omega\,.$$

Solve the following problems.

(1) Obtain the Fourier transform $F(\omega)$ of the following function

$$f(x) = \begin{cases} -x & (0 < x \leq 1) \\ x-2 & (1 < x \leq 2) \\ 0 & (x \leq 0,\ x > 2)\,. \end{cases}$$

(2) Obtain $|F(\omega)|^2$ using $F(\omega)$ obtained in problem (1).

(3) Evaluate $\displaystyle\int_0^{\infty} \frac{(\cos x - 1)^2}{x^4}\, dx$ using $\displaystyle\frac{1}{2\pi}\int_{-\infty}^{\infty} |F(\omega)|^2\, d\omega = \int_{-\infty}^{\infty} \{f(x)\}^2\, dx\,.$

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt\,.$$

Solve the following problems.

(1) Derive

$$\mathcal{L}\left[\frac{f(t)}{t}\right] = \int_s^{\infty} F(s')\, ds',$$

when $f(t)$ satisfies that $\displaystyle\lim_{s\to\infty}\int_0^{\infty}\frac{f(t)}{t}\, e^{-st}\, dt = 0\,.$

(2) Obtain the Laplace transform of $\sin^2 t\,.$

(3) Obtain the Laplace transform of $\dfrac{\sin^2 t}{t}\,.$

(4) Evaluate $\displaystyle\int_0^{\infty}\frac{e^{-2t}\sin^2 t}{t}\, dt\,.$

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東北大学大学院機械系４専攻入学試験

試験問題冊子
【専門科目　Specialized Subjects】

令和元年８月 28 日 (水)　9:00 ― 12:00
Wednesday, August 28, 2019　9:00 ― 12:00

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Select two subjects from the five subjects in the booklet and answer all problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

熱 力 学　THERMODYNAMICS

1. Figure 1 shows gas-liquid equilibrium lines (evaporation curves) including both end points of pure substances A and B plotted on a $\ln p - \frac{1}{T}$ diagram. Here, p and T denote pressure and temperature, respectively, and ln represents natural logarithm. Answer the following questions.

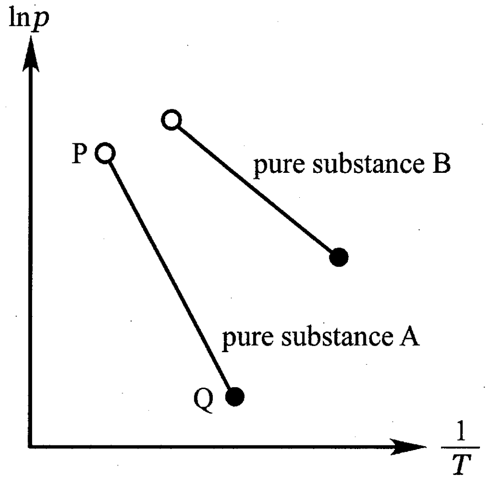

Fig. 1

(1) Describe the names of end points P and Q.

(2) When the pressure and temperature change with keeping the state of gas-liquid equilibrium, show that the following equation is satisfied. Here, specific entropy and specific volume of the gas phase are denoted by s_G and v_G, respectively, and those of the liquid phase are denoted by s_L and v_L, respectively.

$$\frac{dp}{dT} = \frac{s_G - s_L}{v_G - v_L}$$

(3) Express $\frac{dp}{dT}$ in question (2) using v_G, v_L, T and r, where r is the latent heat of vaporization per unit mass.

(4) When the molecular weights of pure substances A and B are equal, show which pure substance has the larger latent heat of evaporation per unit mass, and describe its reason. Assume that the specific volumes of the gas phases are sufficiently large compared with those of liquid phases, and that the gas phases obey the equation of state of an ideal gas. Also, the latent heats of vaporization per unit mass are assumed to be constant.

熱 力 学　THERMODYNAMICS

2. Consider a cycle for an ideal gas in a closed system. The cycle consists of three quasi-steady processes, which are an adiabatic compression process of state 1→2, an isobaric heating process of state 2→3 and an isochoric cooling process of state 3→1. The temperature and specific entropy at state 1 are T_1 and s_1, respectively. The temperature at state 2 is T_2. The temperature and specific entropy at state 3 are T_3 and s_3, respectively. The specific heat ratio and the gas constant of the ideal gas are κ and R, respectively. Answer the following questions.

(1) Show the specific heat at constant volume and that at constant pressure of the gas using κ and R.

(2) Draw the temperature–specific entropy (T–s) diagram of the cycle.

(3) Show the change of specific entropy, $s_1 - s_3$, during the process of state 3→1 using T_1, T_3, κ and R.

(4) Show the temperature at state 3, T_3, using T_1, T_2 and κ.

(5) Show the thermal efficiency of the cycle using ε and κ, where ε is the compression ratio during the process of state 1→2.

流 体 力 学　FLUID DYNAMICS

1. Consider a two-dimensional steady potential flow of an inviscid incompressible fluid. The complex potential $W(z)$ of the flow is given by

$$W(z) = Uz - m\log z\,,$$

where U and m are positive real numbers, and log is the natural logarithm. The complex variable z is given by $z = x + iy = re^{i\theta}$, where x and y are the Cartesian coordinates, r and θ are the radial and circumferential coordinates, respectively, and $i = \sqrt{-1}$. Answer the following questions.

(1) Obtain the velocity potential $\Phi(r,\theta)$ and the stream function $\Psi(r,\theta)$ of the flow field.
(2) Obtain the coordinates of a stagnation point in the flow field.
(3) Obtain the coordinates of the points at which the streamlines through the stagnation point cross the y axis.
(4) Draw streamlines with their directions in the flow field.
(5) Obtain the stagnation pressure, and draw the pressure profile along the x axis, assuming that the pressure far upstream is p_∞ and the density of the fluid is constant at ρ.

流 体 力 学　FLUID DYNAMICS

2. Consider coaxial cylinders centered at the origin O, as shown in Fig. 1. The inner cylinder with the radius a is stationary. The outer cylinder with the radius b is rotating at the constant angular velocity Ω. A steady two-dimensional circumferential flow of an incompressible viscous fluid is induced between the two cylinders. Assuming that external forces are negligible, the density ρ and the viscosity μ of the fluid are constant, answer the following questions.

(1) In the cylindrical coordinate $(r,\ \theta)$ shown in Fig. 1, the radial component of the steady Navier–Stokes equations is expressed as

$$u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r} + \frac{\mu}{\rho}\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2}\right)$$

and the circumferential component is expressed as

$$u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \frac{\mu}{\rho}\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right).$$

Here, p, u_r and u_θ are the pressure, the radial component of the velocity and the circumferential component of the velocity, respectively. Assuming that u_r is zero and that u_θ and p are functions of r alone, answer the following questions.

a) Obtain the relation among r, ρ, p and u_θ using the equation for the radial component.

b) Obtain the relation between u_θ and r using the equation for the circumferential component.

(2) The equation obtained in a) of question (1) indicates that pressure gradient is generated when $u_\theta \neq 0$. Explain the reason of the pressure gradient briefly.

(3) Express u_θ using a, b, Ω and r with the aid of the equation obtained in b) of question (1). Here, particular solutions of the equation have a form of $u_\theta = Cr^k$, where C and k are constants.

(4) Express the shear stress τ exerted on the surface of the inner cylinder using a, b, Ω and μ.

(5) Express the torque T per unit axial length acting on the surface of the inner cylinder using a, b, Ω and μ.

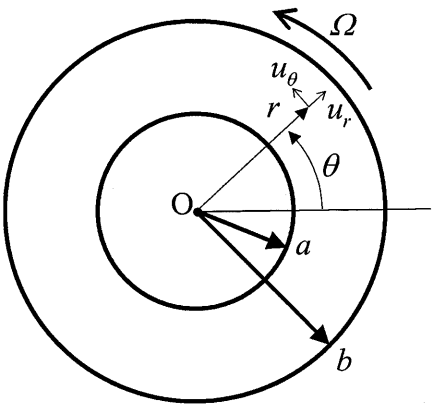

Fig. 1

材 料 力 学　STRENGTH OF MATERIALS

1. Consider a stepped solid circular shaft ABC as shown in Fig. 1. The diameter and length of portion AB are $2d$ and L, and those of portion BC are d and L, respectively. The shaft is fixed at left end A to a vertical rigid wall. The shear modulus of the shaft is G. Answer the following questions.

(1) As shown in Fig. 1(a), the shaft is twisted by the twisting moment M_{t1} at position B and the twisting moment M_{t2} at right end C. Here the twisting direction of M_{t2} is reverse to that of M_{t1}. Find the twisting angle ϕ_C at right end C.

(2) Determine the maximum shear stress and indicate its location in question (1).

(3) As shown in Fig. 1(b), two bars are pin-connected to the shaft surface at two intersection points D and F where a horizontal line passing through the center of the axis cuts the outer circumference of right end C. The length, cross sectional area and Young's modulus of the two bars are h, S and E, respectively. The lower ends of the bars are perpendicularly pin-connected to a rigid horizontal floor. The shaft is twisted by the twisting moment M_t at position B. Find the reactive twisting moment M_C and the twisting angle ϕ_C at right end C.

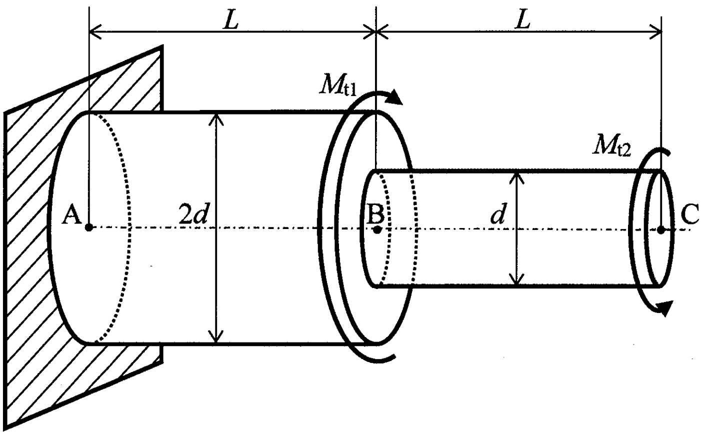

Fig. 1(a)

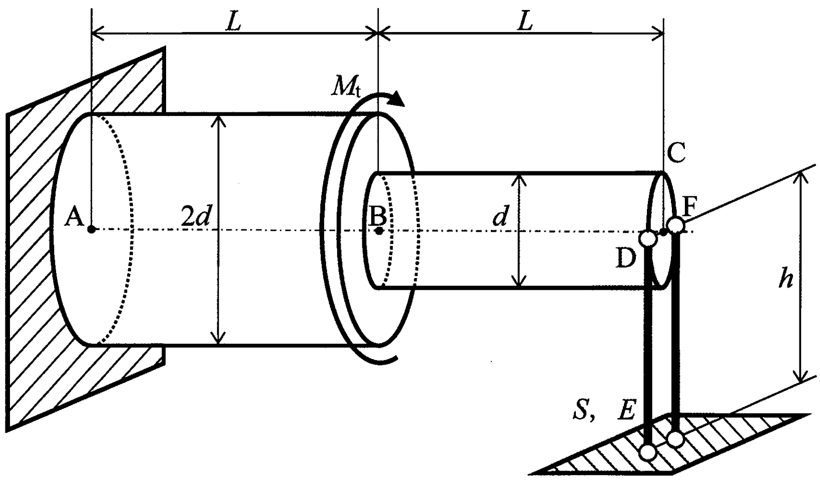

Fig. 1(b)

2. As shown in Fig. 2(a), top view, consider beams A_1B_1, A_2B_2 and CD which are fixed to vertical rigid walls at ends A_1, A_2 and D, respectively, and have holes at ends B_1, B_2 and C. As shown in Fig. 2(b), side view, the difference in the height between the neutral axis of beams A_1B_1 and A_2B_2 and that of beam CD is d, where $d << L$. The length of beams A_1B_1 and A_2B_2 is $2L$, the length of beam CD is L, and flexural rigidity EI of beams A_1B_1, A_2B_2 and CD is constant. A vertical load is applied to beam CD at end C until the centers of all the holes are aligned on a straight line, and then, a pin is inserted through the holes. When the load is released, beams A_1B_1, A_2B_2 and CD are deflected to an equilibrium state. Assume that the diameters of the pin and the holes are identical, and the deformations and friction of the pin and the holes are negligible. Neglect both the weight of the beams and the elongation of the neutral axis of the beams. Answer the following questions.

(1) Determine the deflection of the beams, δ, at ends B_1 and B_2.
(2) Determine the deflection angle of beam A_1B_1, ϕ_{B_1}, at end B_1 and that of beam CD, ϕ_C, at end C, respectively.
(3) Determine the reaction force of the beam, R_D, at end D.
(4) Determine the reaction moment of the beam, M_D, at end D.

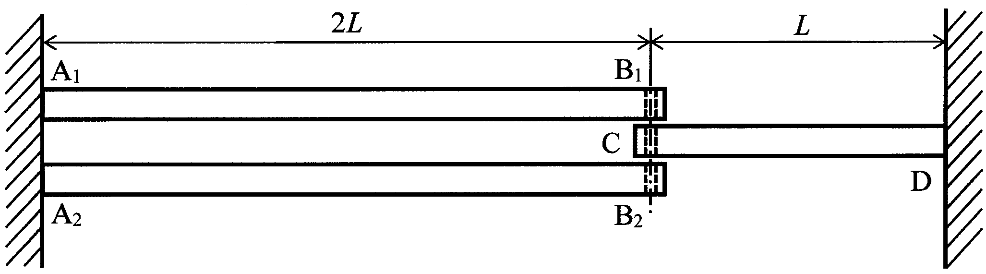

Fig. 2(a) Top view

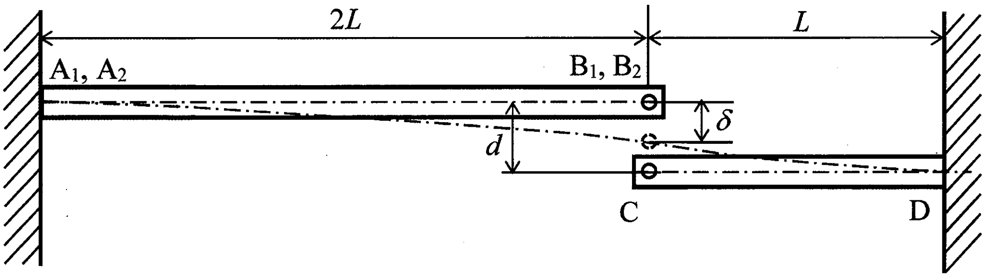

Fig. 2(b) Side view

機械力学 DYNAMICS OF MECHANICAL SYSTEMS

1. Consider a system consisting of a uniform rigid bar with mass m and length $a + b$, two springs with spring constants k and $2k$, and a dashpot with damping coefficient c, as shown in Fig. 1. The bar is pinned at point O, and rotationally vibrates with sufficiently small displacement around equilibrium position in the plane of the figure. The angular displacement of the rigid bar from the equilibrium position is denoted by $\theta(t)$, where t is time. The dashpot is connected to a ceiling and the ceiling is subjected to vertical displacement excitation $u(t) = A \sin \omega t$, where A and ω are the amplitude and the angular frequency of the displacement, respectively. Assume that the masses of the springs and the dashpot are negligible. When the system is in the steady-state, answer the following questions.

   (1) Obtain the moment of inertia J of the bar about point O.

   (2) Derive the equation of motion of the system.

   (3) When $c = 0$, $a = 2\ell$, $b = \ell$, find the natural angular frequency of the system.

   (4) When $c \neq 0$, $a = 2\ell$, $b = \ell$, find the angular displacement $\theta(t)$.

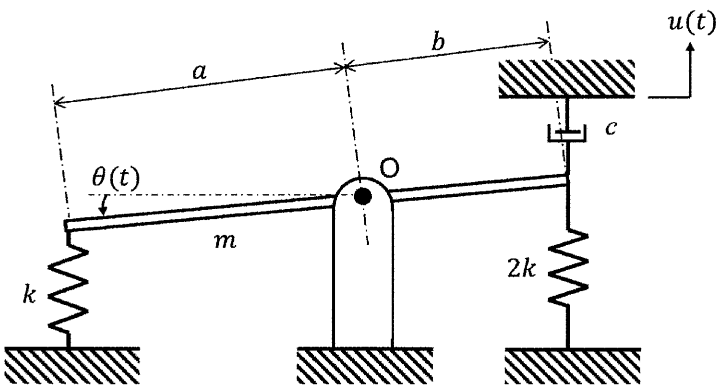

Fig. 1

2. Consider a system consisting of two pulleys with radius r_1, r_2 and mass moments of inertia J_1, J_2, and three springs with spring constants k_1, k_2 and k_3, as shown in Fig. 2. The axes of two pulleys are fixed and the three springs are connected by ropes. The ropes do not loosen and there is no slip between the ropes and the pulleys. The masses of the ropes and the springs are negligible. The angular displacements of the pulleys from the equilibrium positions are denoted by θ_1 and θ_2. Answer the following questions.

(1) Derive the equations of motion of the system.

(2) Derive the frequency equation of the system.

(3) When $J_1 = J$, $J_2 = 4J$, $r_1 = r$, $r_2 = 2r$, and $k_1 = k_2 = k_3 = k$, find the natural angular frequencies of the system.

(4) Find the amplitude ratio of θ_2 to θ_1 at each natural angular frequency obtained in question (3).

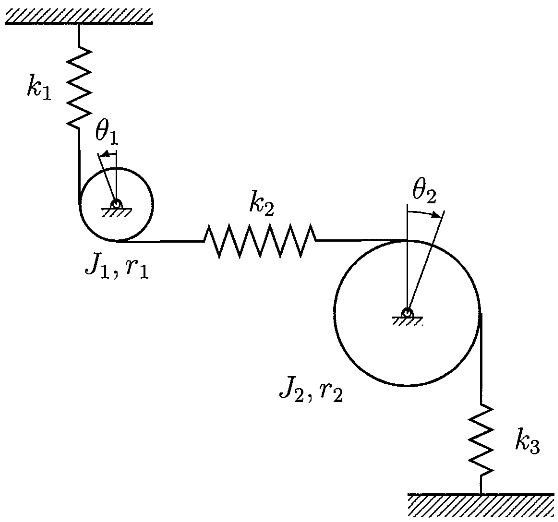

Fig. 2

制御工学 CONTROL ENGINEERING

1. Solve the following problems.

(1) Find the impulse response of the transfer function

$$G(s) = \frac{7s+15}{s(s+3)(s+5)}.$$

(2) Consider the control system shown in Fig. 1.

a) Find the range of k for the system to be stable.

b) Draw the root loci of the system.

c) Calculate the gain margin when $k = 16$.

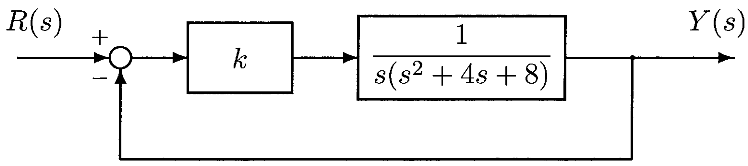

Fig. 1

(3) When the vector locus of the frequency transfer function $G(j\omega)$ is given in Fig. 2, find the output for the input $\sin(\omega_0 t)$.

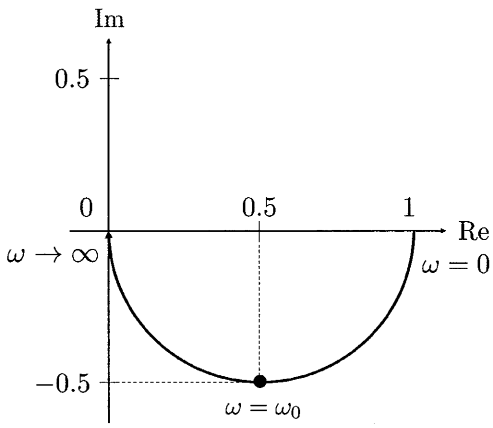

Fig. 2

(4) Consider the system given in Fig. 3. When the disturbance is $d(t) = t$ and the input is $r(t) = 0$, determine the range of k for the steady-state output to be $|y(\infty)| < 0.5$.

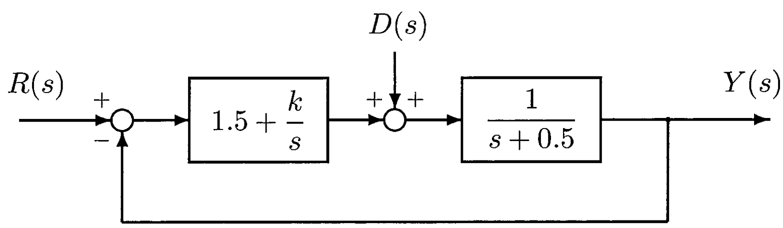

Fig. 3

2. Solve the following problems.

(1) Consider the system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t), \quad y(t) = \boldsymbol{c}\boldsymbol{x}(t),$$

where $u(t)$ is the input, $y(t)$ is the output, and $\boldsymbol{x}(t) = [x_1,\ x_2]^T$ is the state vector. When

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \boldsymbol{c} = [1,\ 2],$$

determine the range of $\boldsymbol{k} = [k_1,\ k_2]$ for the closed-loop system to be stable under the state feedback $u(t) = -\boldsymbol{k}\boldsymbol{x}(t)$.

(2) Suppose $\boldsymbol{k} = [9,\ 7]$ in problem (1). Find the coordinate transformation $\boldsymbol{x}(t) = \boldsymbol{T}\boldsymbol{z}(t)$ to diagonalize the closed-loop system, and find the diagonalized system $\dot{\boldsymbol{z}}(t) = \boldsymbol{M}\boldsymbol{z}(t)$.

(3) Consider the system

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{F}\boldsymbol{z}(t) + \boldsymbol{g}v(t), \quad y(t) = \boldsymbol{h}\boldsymbol{z}(t),$$

where $v(t)$ is the input, $y(t)$ is the output, and $\boldsymbol{z}(t) = [z_1,\ z_2]^T$ is the state vector. When

$$\boldsymbol{F} = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}, \quad \boldsymbol{g} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad \boldsymbol{h} = [-1,\ -3],$$

and the initial state $\boldsymbol{z}(0) = [0,\ 0]^T$, obtain the response for the unit step input.

(4) For the system in problem (3), consider the error integral feedback $v(t) = \lambda\xi(t)$ as shown in Fig. 4, where λ is a constant gain, $r(t)$ is the input, and $y(t)$ is the output.

a) Derive the state equation of the augmented system with the state vector as $[z_1,\ z_2,\ \xi]^T$.

b) Find the range of λ for the system to be stable.

c) Evaluate the steady-state error $e(\infty)$ for the unit step input.

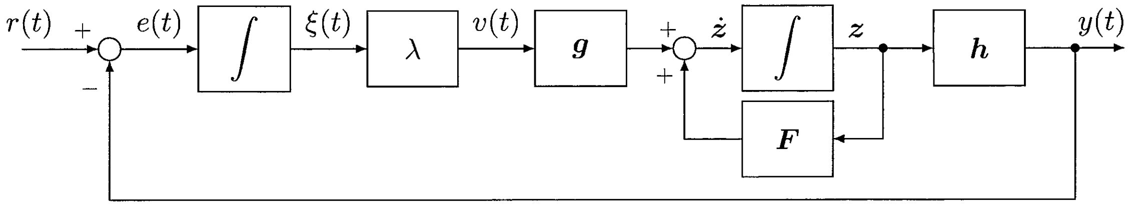

Fig. 4