英

令和２年度 秋季募集
(令和３年４月入学)
東北大学大学院機械系４専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

令和２年８月25日(火)
Tuesday, August 25, 2020　　9:30 － 10:30

Notice

1. Do not open this test booklet until instructed to do so.

2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets, the draft sheets.

3. Answer all problems. Use two answer sheets for each problem.

4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学 A　MATHEMATICS A

1. Solve the following problems.

   (1) Evaluate the definite integral.

$$\int_0^{\frac{\pi}{4}} \frac{2x\sin x}{\cos^3 x}\,dx$$

   (2) Evaluate the indefinite integral.

$$\int \frac{2x^2-4x-2}{x^4-2x^2+1}\,dx$$

   (3) Evaluate the integral.

$$\int_D \sqrt{x^2+y^2}\,dxdy, \qquad D=\{\,(x,y)\mid x^2+y^2\leq 2x\,\}$$

2. Consider the quadratic form in an xy plane given by

$$f(x,y)=\frac{3}{2}x^2-\sqrt{3}xy+\frac{5}{2}y^2.$$

Let A be the symmetric matrix that satisfies

$$f(x,y)=(x\ \ y)\,A\begin{pmatrix}x\\y\end{pmatrix}.$$

Solve the following problems.

   (1) Obtain A and its eigenvalues.

   (2) Obtain one of the orthogonal matrices which diagonalize A.

   (3) Obtain the maximum and minimum values of $f(x,y)$ under $x^2+y^2=1$.

3. In the Cartesian coordinate system (x, y, z), the vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = z\,\boldsymbol{i} + e^z\,\boldsymbol{j},$$

where the fundamental vectors in the x, y, and z directions are denoted by $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$, respectively. Let S be the surface given by

$$S:\ z = \sqrt{x^2 + y^2}, \quad x \geq 0, \quad y \geq 0, \quad 0 \leq z \leq 1.$$

Solve the following problems.

(1) Obtain $\nabla \times \boldsymbol{A}$.

(2) Obtain the unit normal vector $\boldsymbol{n}$ of S, where the z component of $\boldsymbol{n}$ is positive.

(3) Evaluate the integral

$$\int_S (\nabla \times \boldsymbol{A}) \cdot \boldsymbol{n}\, dS,$$

where $\boldsymbol{n}$ is the unit normal vector of S obtained in problem (2).

英

令和 2 年度 秋季募集
(令和 3 年 4 月入学)
東北大学大学院機械系 4 専攻入学試験

試験問題冊子

数学Ｂ　MATHEMATICS B

令和 2 年 8 月 25 日(火)
Tuesday, August 25, 2020　　13:30 － 14:30

Notice

1．Do not open this test booklet until instructed to do so.

2．A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets, the draft sheets.

3．Answer all problems. Use two answer sheets for each problem.

4．At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\left(x^2 + y^2 e^{\frac{x}{y}}\right) \dfrac{dy}{dx} = xy$

(2) $2\dfrac{d^2y}{dx^2} - 11\dfrac{dy}{dx} + 12y = \sinh x$

2. The function $u(x, y)$ satisfies the partial differential equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \qquad (0 < x < a,\ 0 < y < b)$$

with the boundary conditions

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0, \qquad \left.\frac{\partial u}{\partial x}\right|_{x=a} = 0, \qquad \left.\frac{\partial u}{\partial y}\right|_{y=0} = 0,$$

where $u(x, y)$ is not a constant function. Solve the following problems.

(1) Obtain the ordinary differential equations for $P(x)$ and $Q(y)$ when $u(x, y) = P(x)Q(y)$.

(2) Obtain the general solutions of $P(x)$ and $Q(y)$ in problem (1).

(3) The function $u(x, y)$ satisfies the boundary condition $u(x, b) = \cos\left(\dfrac{2\pi}{a}x\right)$. Obtain $u(x, y)$ by using the general solutions of problem (2).

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt\,.$$

Solve the following problems.

(1) Obtain the inverse Laplace transform of $\dfrac{d}{ds}F(s)$.

(2) Obtain the Laplace transform of $t\cos(\omega t)$.

(3) Obtain the inverse Laplace transform of $\dfrac{\omega^2}{(s^2+\omega^2)^2}$.

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試験問題冊子
【専門科目　Specialized Subjects】

令和２年８月 26 日 (水)　9:30 － 10:30
Wednesday, August 26, 2020　9:30 － 10:30

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Select two subjects from the five subjects in the booklet and answer all problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

熱 力 学　THERMODYNAMICS

1. Answer the following questions.

   (1) Express the Clapeyron-Clausius equation and explain meanings of symbols used.

   (2) By introducing several assumptions into the Clapeyron-Clausius equation, saturated vapor pressure p of a pure substance at temperature T is derived as follows:

   $$\ln p = A - B / T$$

   where A and B are constants, and ln represents natural logarithm. Describe all the assumptions introduced in this derivation.

熱 力 学　THERMODYNAMICS

2. A gas turbine which employs an ideal gas as the working fluid operates with a Brayton cycle. The states of the inlet and outlet of the compressor are defined as state 1 and state 2, respectively. The states of the inlet and outlet of the turbine are defined as state 3 and state 4, respectively. The temperature of each state is defined as T_i (i = 1, 2, 3 and 4). For this Brayton cycle, answer the following questions.

(1) Draw a pressure–specific volume (p–v) diagram and a temperature–specific entropy (T–s) diagram. Indicate states 1, 2, 3 and 4 in each diagram.

(2) Derive $T_1 / T_2 = T_4 / T_3$.

(3) In the case of T_4 = 800 K, find T_3 that gives the theoretical thermal efficiency of 50%.

1. As shown in Fig. 1, an L-shaped pipe is fixed in space and partially immersed in water. The water surface is horizontal, and the water flows parallel to the surface with uniform velocity U. The L-shaped pipe consists of the horizontal part and the vertical part of length $3b$, which is parallel to the gravitational direction. The cross-sectional area of the horizontal part is $2A$, and that of the vertical part is A. The horizontal part is immersed at depth b from the water surface, and the vertical axis z is taken upward from the center axis of the horizontal pipe at $z = 0$. The pipe diameters are assumed to be sufficiently small compared with the water depth b and the horizontal pipe length. The density of water is ρ, and the gravitational acceleration is g. The pipe loss, viscosity and surface tension are negligible. Answer the following questions.

(1) Consider the case when the water column in the vertical part has a steady height at $z = h$, where $b < h < 3b$. Express h using U, b and g.

(2) Consider the case when the water is ejected from the top of the vertical part with steady velocity V.
   a) Express the ejection velocity V using U, b and g.
   b) Derive the horizontal force to fix the L-shaped pipe in space by using V, A and ρ. Here, the drag acting on the outer surface of the L-shaped pipe is negligible.

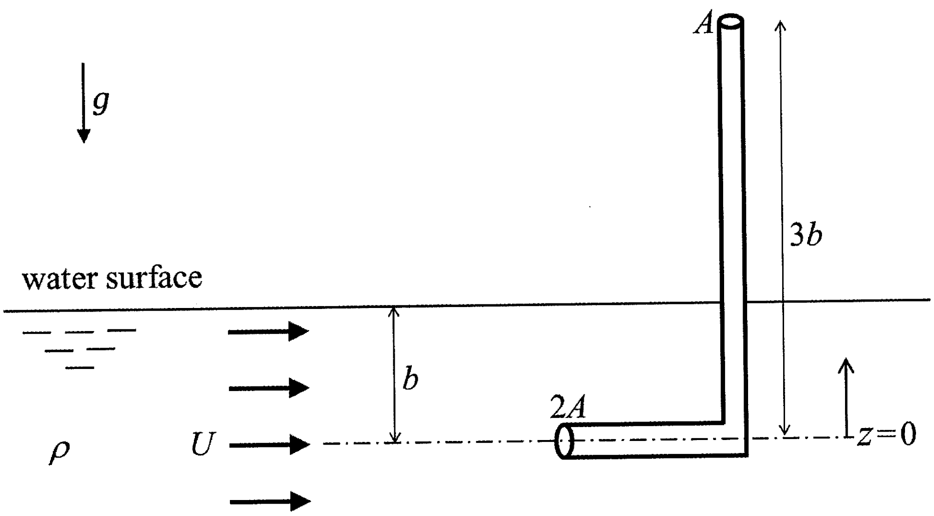

Fig. 1

2. Consider a two-dimensional steady flow of an inviscid incompressible fluid in the x-y plane. The velocity potential $\phi(x, y)$ of this flow is given by

$$\phi(x, y) = 3x^2 + (2y + 1)x - 3y^2 - 2y .$$

Answer the following questions.

(1) Express the velocity component u in the x-direction and the velocity component v in the y-direction using x and y.
(2) Show that u and v satisfy the continuity equation.
(3) Show that the flow is irrotational by using the definition of vorticity.
(4) Obtain the stream function $\psi(x, y)$, where $\psi(0, 0) = 0$.

材 料 力 学　STRENGTH OF MATERIALS

1. As shown in Fig. 1, an L-shaped cantilever beam ABC of a circular cross section is fixed horizontally to a rigid wall at A. The lengths of AB and BC are a and b, respectively. The torsional rigidity and flexural rigidity of the L-shaped beam are GI_P and EI, respectively. Assume that the weight of the beam is negligible. Answer the following questions.

(1) As indicated in Fig. 1(a), a twisting moment T_1 is applied around the axis of the portion AB at B in the direction depicted in the figure. Draw the distribution diagram of the internal force in the portion AB. And find the vertical deflection of the beam at C.

(2) As illustrated in Fig. 1(b), a twisting moment T_2 is applied around the axis of the portion BC at C in the direction indicated in the figure in addition to the twisting moment T_1 given in question (1). Draw the distribution diagrams of the internal forces in the portions AB and BC, respectively.

(3) Determine the angle of twist of the beam at C under the loading condition of question (2).

(4) Calculate the vertical deflection of the beam at C under the loading condition of question (2).

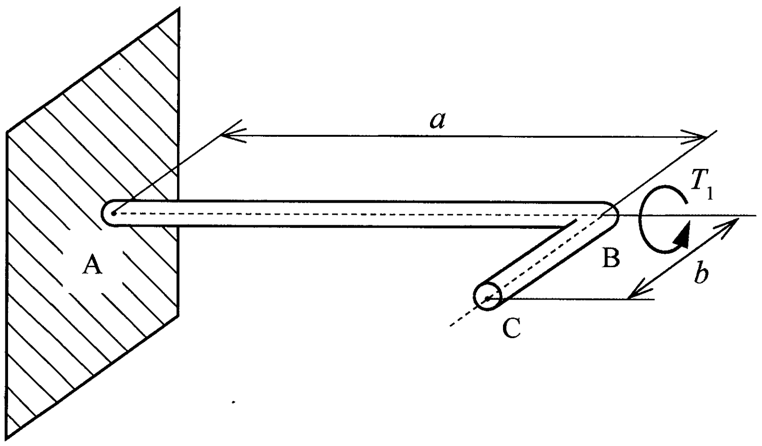

Fig. 1(a)

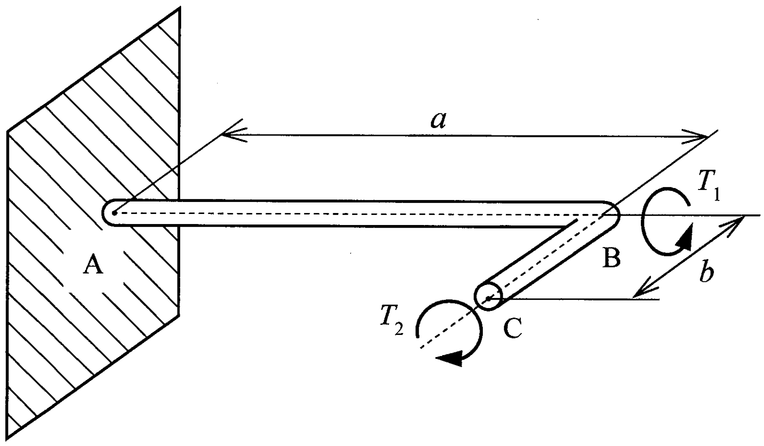

Fig. 1(b)

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機械力学 DYNAMICS OF MECHANICAL SYSTEMS

1. Consider a system consisting of a mass m, a massless rigid bar with length ℓ, a spring with spring constant k, and a dashpot with damping constant c, as shown in Fig. 1. One end of the rigid bar is pinned at a wall at point O, and mass m is connected to the other end. One end of the parallel combination of the spring and the dashpot is connected to the rigid bar at point A, and the other end is fixed at a ceiling. The distance from point O to point A is r. The ceiling is subjected to vertical displacement excitation $u(t) = a \sin \omega t$, where a and ω are the amplitude and angular frequency of the displacement, respectively. t is time. The rigid bar is horizontal at the equilibrium position. The angular displacement of the rigid bar from the equilibrium position is denoted by θ. Assuming that θ is sufficiently small, answer the following questions.

    (1) Obtain the change of the length of the spring.

    (2) Obtain the force applied to the rigid bar at point A via the parallel combination of the spring and the dashpot.

    (3) Derive the equation of motion of the system.

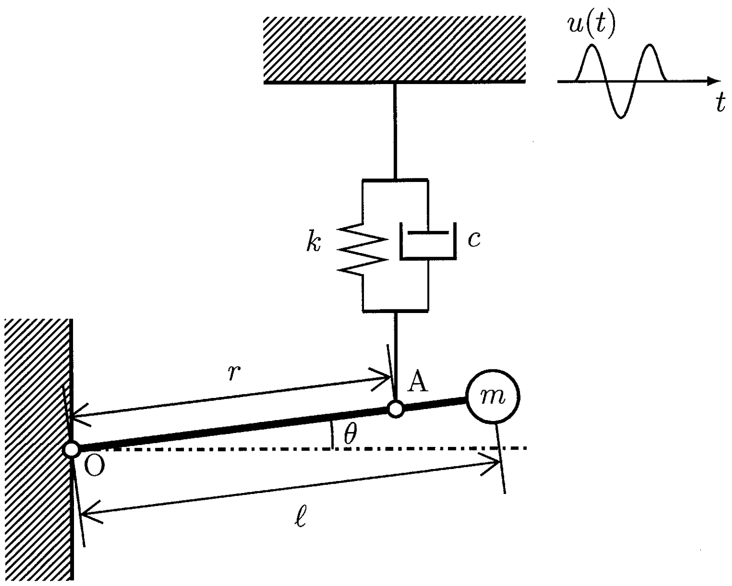

Fig 1.

2. Consider a system consisting of two masses m_1 and m_2, and three springs with spring constants k_1, k_2 and k_3. The displacements of masses m_1 and m_2 from the equilibrium positions are denoted by x_1 and x_2, respectively. Assuming that the masses of the springs are negligible, the equations of motion of the system are represented as follows:

$$m_1\ddot{x}_1 = -k_1x_1 + k_2(x_2 - x_1) \ ,$$

$$m_2\ddot{x}_2 = -k_2(x_2 - x_1) - k_3x_2 \ .$$

Answer the following questions.

(1) When $m_1 = m_2 = m$, $k_1 = k_3 = k$, and $k_2 = 2k$, obtain the natural angular frequencies of the system.

(2) Obtain the amplitude ratio of x_1 to x_2 at each natural angular frequency obtained in question (1).

制御工学 CONTROL ENGINEERING

1. Solve the following problems.

(1) Consider a system shown in Fig.1. $R(s)$ and $Y(s)$ are the Laplace transoforms of the reference $r(t)$ and the output $y(t)$. The plant transfer function $P(s)$ and the controller transfer function $C(s)$ are given by

$$P(s) = \frac{10}{s(s+10)}, \quad C(s) = \frac{k}{s+1},$$

respectively. k is a constant.

a) When $k = 10$, find the coordinate where the vector locus of the open-loop system intersects the real axis.

b) Find the conditions on k to stabilize the control system.

c) Draw the root loci for $k \in [0, \infty)$.

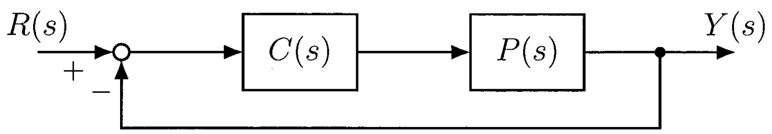

Fig. 1

制御工学 CONTROL ENGINEERING

2. Solve the following problems.

(1) Consider the system expressed by a differential equation

$$\ddot{x}(t) - 2\dot{x}(t) - 3x(t) = au(t),$$

where a is a constant.

a) Describe the state equation of this system in terms of the state vector $\boldsymbol{x}(t) = \begin{bmatrix} x(t) & \dot{x}(t) \end{bmatrix}^T$ and the input $u(t)$.

b) Find the condition on a for this system to be uncontrollable, and explain the reason that the system becomes uncontrollable under this condition.

c) Suppose that $u(t) = 0$. Obtain the solution $x(t)$ of this differential equation for initial values $x(0) = 1$ and $\dot{x}(0) = 0$.

(2) Consider the system expressed by a state equation

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} 0 & 1 \\ 5 & 4 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t),$$

where $\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$ is the state vector and $u(t)$ is the input. Suppose that a state feedback $u(t) = -\boldsymbol{k}^T \boldsymbol{x}(t)$ is applied to this system. Find the condition on $\boldsymbol{k}^T = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ to stabilize this closed-loop system.