英

令和４年度 秋季募集
東北大学大学院機械系４専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

令和 4 年 8 月 30 日(火)
Tuesday, August 30, 2022　　9:30 − 10:30 (JST)

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets, the draft sheets.
3. Answer all problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

For online written test, please follow the examiner's instructions.

数 学 A　MATHEMATICS A

1. Solve the following problems.

(1) Evaluate the following integral

$$\int \frac{1}{\cosh x + 1}\, dx.$$

(2) Evaluate the following limit

$$\lim_{x \to 0} \frac{x \cos x - \log_e (1 + x)}{x \sin x}.$$

(3) Evaluate the following integral

$$\iint_D \sqrt{\frac{1 - x^2 - y^2}{1 + x^2 + y^2}}\, dxdy,$$

where the region D is given by

$$D = \{\ (x, y) \mid x^2 + y^2 \leq 1\ \}.$$

数学A　MATHEMATICS A

2. Consider n points $P_1, P_2, \ldots,$ and P_n, and some pairs of them are connected by line segments. The components a_{ij} of the corresponding $n \times n$ matrix are defined as

$$a_{ij} = \begin{cases} 0 & i = j, \\ 1 & i \neq j;\ P_i \text{ and } P_j \text{ are connected}, \\ 0 & i \neq j;\ P_i \text{ and } P_j \text{ are not connected}. \end{cases}$$

For example, the matrices corresponding to Figs. 1 and 2 are

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix},$$

respectively. Solve the following problems.

(1) Find the eigenvalues of B and C.

(2) Obtain the sum of the diagonal components of C^3.

(3) Obtain the sum of the diagonal components of D^3, where D is the matrix corresponding to Fig. 3.

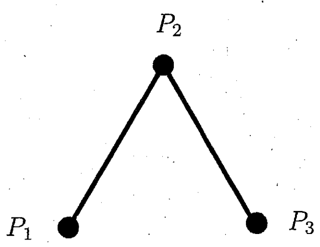

Fig. 1

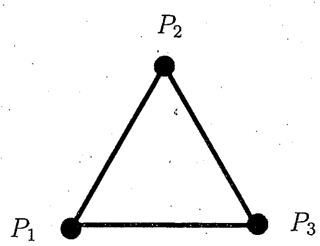

Fig. 2

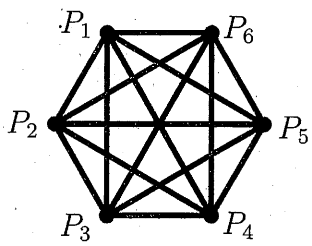

Fig. 3

数学 A　MATHEMATICS A

3. In the Cartesian coordinate system (x, y, z), the vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = (x + yz)\,\boldsymbol{i} + (x + x^2 + y^2 + zx)\,\boldsymbol{j} + (z + xy)\,\boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors in the x, y, and z directions, respectively. The region V is given by

$$V:\ 0 \le z \le 1 - x^2, \quad 0 \le y \le 1.$$

Solve the following problems.

(1) Obtain $\nabla \times \boldsymbol{A}$ and $\nabla \cdot \boldsymbol{A}$.

(2) Evaluate the following surface integral

$$\int_S \boldsymbol{A} \cdot \boldsymbol{n}\, dS,$$

where S is the surface of V and $\boldsymbol{n}$ is the unit outward normal vector of S.

(3) Let S' be the part of the surface of V given by

$$S':\ z = 1 - x^2, \quad 0 \le y \le 1, \quad z \ge 0.$$

Evaluate the following line integral

$$\int_{\partial S'} \boldsymbol{A} \cdot d\boldsymbol{r},$$

where $\partial S'$ is the boundary of S', $\boldsymbol{r}$ is the position vector on $\partial S'$, and the line integral is taken in the rotation direction of the right-handed screw advancing in the $+z$ direction.

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試験問題冊子

数学B　MATHEMATICS B

令和４年８月30日(火)
Tuesday, August 30, 2022　13:00 － 14:00 (JST)

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets, the draft sheets.
3. Answer all problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

For online written test, please follow the examiner's instructions.

数学 B MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\dfrac{dy}{dx} = -\dfrac{2(x^4+1)(y^2-1)}{xy}$

(2) $\left(\dfrac{dy}{dx}\right)^2 + (x-3y-1)\dfrac{dy}{dx} + 2y^2 - 2xy - x + y = 0$

2. Solve the following problems.

(1) Obtain the Fourier series of the function $f(x)$ with period 2π given by

$$f(x) = x^2 \qquad (-\pi \leq x < \pi).$$

(2) Obtain the Fourier series of the function $g(x)$ with period 2π given by

$$g(x) = \begin{cases} x(x-2\pi) & (0 \leq x < \pi) \\ x(x+2\pi) & (-\pi \leq x < 0). \end{cases}$$

数学 B　MATHEMATICS B

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt .$$

For any continuous function $\phi(t)$, the delta function $\delta(t)$ satisfies

$$\int_{-\infty}^{\infty} \phi(t)\delta(t)dt = \phi(0) .$$

The Laplace transform of $\delta(t)$ is $\mathcal{L}[\delta(t)] = 1$. When T is a positive constant, solve the following problems.

(1) Obtain the Laplace transform of the function $x(t)$ defined by

$$x(t) = \begin{cases} 1 & (0 \le t < T) \\ -1 & (T \le t < 2T) \\ 0 & (t < 0,\ t \ge 2T) . \end{cases}$$

(2) The function $g(t)$ is defined by

$$g(t) = \sum_{n=0}^{\infty} \delta(t - nT) .$$

Show that $\mathcal{L}[g(t)] = \dfrac{1}{1 - e^{-sT}}$.

(3) Obtain the function $y(t)$ defined by

$$y(t) = \int_0^t x(u) g(t-u) du ,$$

where $x(t)$ and $g(t)$ are given in problems (1) and (2), respectively.

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試験問題冊子
【専門科目　Specialized Subjects】

熱力学	THERMODYNAMICS	P1~P2
流体力学	FLUID DYNAMICS	P3~P4
材料力学	STRENGTH OF MATERIALS	P5~P6
機械力学	DYNAMICS OF MECHANICAL SYSTEMS	P7~P8
制御工学	CONTROL ENGINEERING	P9~P10

令和４年８月30日（火）
Tuesday, August 30, 2022　　15:45 － 16:45（JST）

Notice

1. Do not open this test booklet until instructed to do so.

2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.

3. Select two subjects from the five subjects in the booklet and answer all problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.

4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

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熱力学 THERMODYNAMICS

1. Consider the quasi-static process of an ideal gas of 1 kg from state 1 to state 2. The temperatures, pressures and specific volumes of this ideal gas at state 1 and state 2 are T_1 and T_2, p_1 and p_2, v_1 and v_2, respectively. The specific heat at constant pressure, specific heat at constant volume, specific heat ratio and gas constant are c_p, c_v, κ and R, respectively. Answer the following questions.

(1) When this process is an isobaric process, show absolute work, heat from outside, and specific enthalpy change using the necessary symbols from c_p, c_v, T_1 and T_2.

(2) When this process is an isochoric process, derive the mathematical condition in the case that technical work is positive.

(3) When a polytropic index is n, absolute work of the polytropic process l_a is expressed by the following equation

$$l_a = \frac{R}{n-1}(T_1 - T_2).$$

Derive the polytropic index n under the condition that absolute work of the polytropic process l_a is equal to the technical work of adiabatic expansion process l_{12}.

熱 力 学 THERMODYNAMICS

2. The left side of Fig. 1 shows a pressure-specific volume (p-v) diagram in which the saturated liquid line and the dry saturated vapor line of a certain material in a temperature range from the triple-point temperature T_T to the critical point temperature T_C are indicated. The right side of Fig. 1 shows a blank pressure-temperature (p-T) diagram. Δv denotes the difference in a specific volume between the saturated liquid and the dry saturated vapor at temperature T. Δs denotes the difference in a specific entropy. Answer the following questions.

(1) Draw the diagrams of Fig. 1 on your answer sheet and answer the following questions.

a) Draw the isothermal lines of temperatures T_C and T_T on the p-v diagram on your answer sheet.

b) Show the regions of compressed liquid and superheated vapor on the p-v diagram on your answer sheet.

c) Draw the above two isothermal lines, the saturated liquid line and the dry saturated vapor line on the p-T diagram on your answer sheet.

(2) Using the necessary symbols from Δv, Δs and T, show the gradient of the saturated liquid line and the dry saturated vapor line at temperature T in the p-T diagram drawn in question (1), respectively.

(3) For water at temperatures 300 K and 500 K, the differences in specific volume between saturated liquid and dry saturated vapor, Δv, are 39 m^3/kg and 0.075 m^3/kg, respectively, and the latent heats of vaporization L are 2.4×10^3 kJ/kg and 1.8×10^3 kJ/kg, respectively. Calculate the gradients in question (2) at these temperatures.

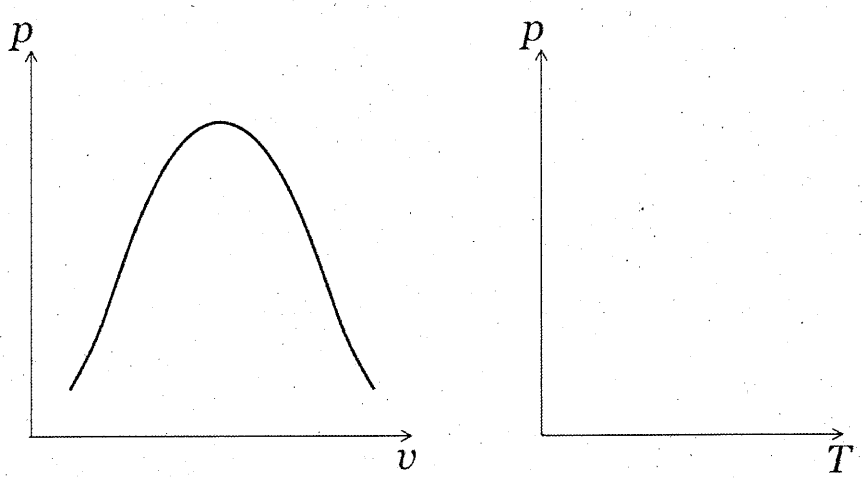


Fig. 1

流 体 力 学　FLUID DYNAMICS

1. Consider a sphere of the density ρ_s and the radius a, falling in an incompressible viscous fluid of the density ρ and the viscosity μ as shown in Fig. 1. Here, C_D is the drag coefficient, V is the terminal velocity of the sphere toward the gravitational direction, and g is the gravitational acceleration. The densities ρ and ρ_s are constant. Answer the following questions.

(1) Obtain the Reynolds number Re of the flow around the sphere using all the symbols of ρ, μ, a and V. Here, the reference length is the diameter of the sphere.

(2) Obtain the drag force acting on the sphere using all the symbols of μ, a and V when $C_D = 24/Re$.

(3) Obtain the terminal velocity V using all the symbols of ρ, ρ_s, μ, a and g, under the condition of question (2).

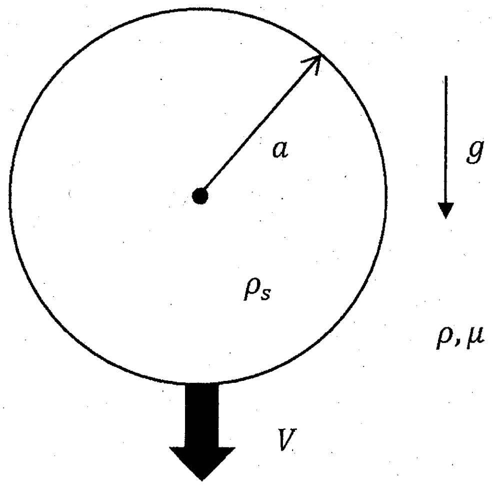


Fig. 1

2. Consider a two-dimensional steady potential flow of an inviscid incompressible fluid as shown in Fig. 2. Here, r and θ are the radial and circumferential coordinates, respectively, and x and y are the Cartesian coordinates. The potential $\phi(r,\theta)$ of the flow is given by the following equation

$$\phi(r,\theta)=U\left(r+\frac{a^2}{r}\right)\cos(\theta-\beta)+\frac{\Gamma\theta}{2\pi},$$

and it specifies the flow around a circular cylinder of the diameter a in the freestream of the velocity U inclined at the angle β ($0<\beta<\pi/2$) from the x axis. Here, U and a are positive real numbers, and circulation Γ is a real number. The density ρ of the fluid is constant. The points A and B in Fig. 2 specify stagnation points. Answer the following questions.

(1) Express the radial velocity v_r and the circumferential velocity v_θ, respectively, by necessary symbols from U, a, β, Γ, r and θ.

(2) Obtain Γ using U, a and β when the position of the stagnation point B is $(r,\theta)=(a,0)$.

(3) Obtain the x- and y-directional components of the fluid dynamic force using ρ, U, a and β, under the condition of question (2). Assume that the fluid force of the magnitude $|\rho U\Gamma|$ in the perpendicular direction of the uniform flow is acted on the object when the circulation Γ exists in the uniform flow and that D'Alembert's paradox stands.

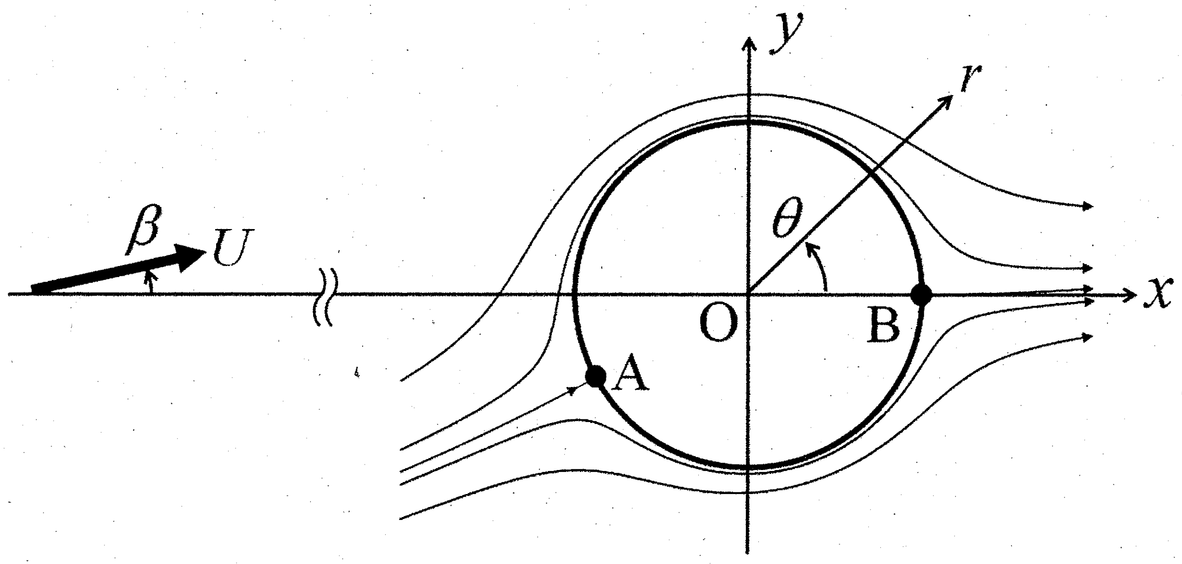


Fig. 2

材 料 力 学　STRENGTH OF MATERIALS

1. A composite solid circular shaft with diameter d, which consists of shafts AB, BC and CD, is fixed to a rigid wall at two ends A and D as shown in Fig. 1(a). The length of shafts AB and CD is L, and their coefficient of thermal expansion, Young's modulus and modulus of rigidity are α_1, E_1 and G_1, respectively. Those of shaft BC are $2L$, α_2, E_2 and G_2, respectively. Assume $\alpha_1 > \alpha_2 > 0$ and $E_1 > E_2 > 0$. Answer the following questions.

(1) The temperature of the shaft is elevated by ΔT. Determine the thermal stress in the shaft.

(2) Determine the deformation of shaft BC and indicate whether it expands or shrinks in question (1).

(3) The composite shaft is subjected to twisting moments $2M$ and M at positions B and C, respectively, as shown in Fig. 1(b). Determine reactive twisting moments M_A and M_D at ends A and D of the shaft, respectively.

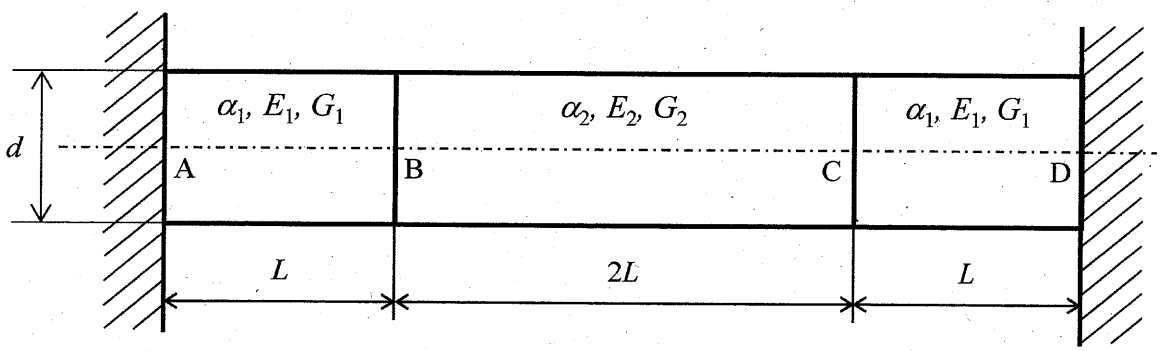


Fig. 1(a)

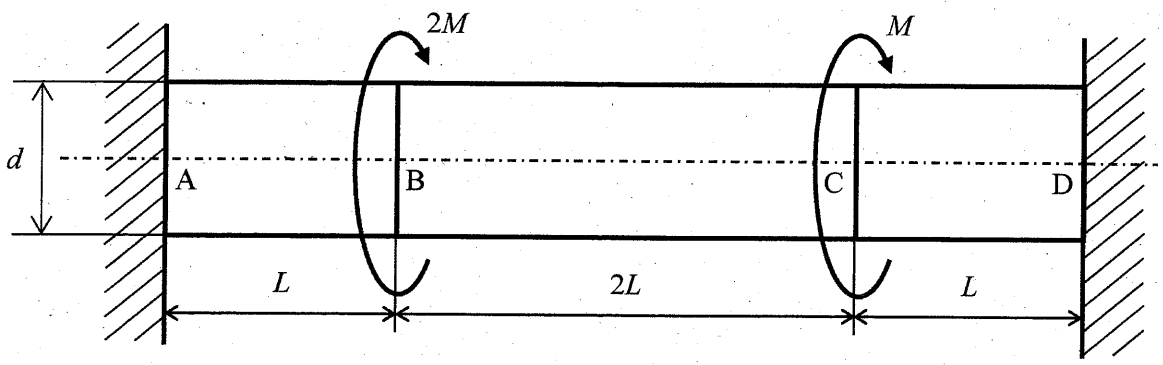
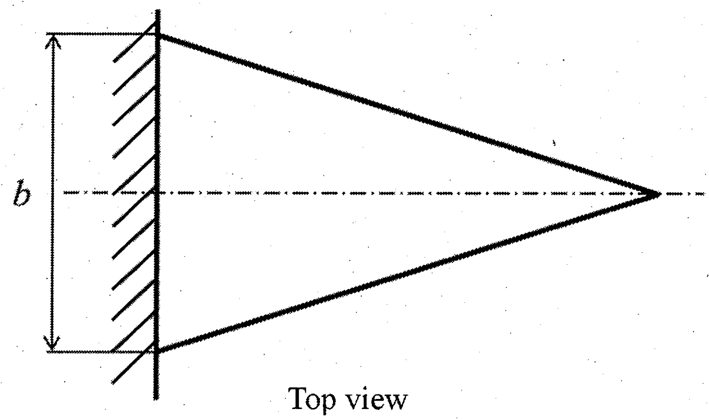


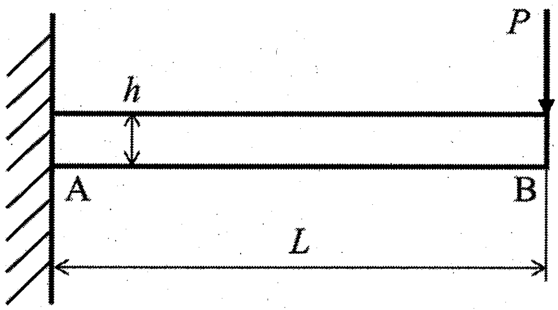
Fig. 1(b)

2. As shown in Fig. 2, a triangular-shaped cantilever beam AB of length L is fixed on a vertical rigid wall at end A. The thickness h of the beam is constant, and its width at end A is b. Young's modulus of the beam is E. A vertical concentrated load P is applied downward at point B. Neglect the weight of the beam. Answer the following questions.

(1) Determine the deflection of beam AB at point B.
(2) Find the magnitude and location of the maximum tensile bending stress in beam AB.
(3) Determine the strain energy stored in beam AB.



Top view



Side view

Fig. 2

1. Consider a torsional system consisting of a disk with mass moment of inertia J and a shaft with rotational stiffnesses k_1, k_2 as shown in Fig. 1. The mass of the shaft is negligible, and the disk can vibrate only in the rotational direction. The disk is subjected to rotational moment excitation $M(t) = A\cos\omega t,$ where A, ω, and t are the amplitude of input moment, the angular frequency and time, respectively. The angular displacement of the disk from the equilibrium position is θ. The response of the system is assumed to have reached the steady state. Answer the following questions.

(1) Derive the equations of motion of the system.
(2) Find the amplitude of the angular displacement of the disk.
(3) Determine the angular frequency ω when the system is in resonance.

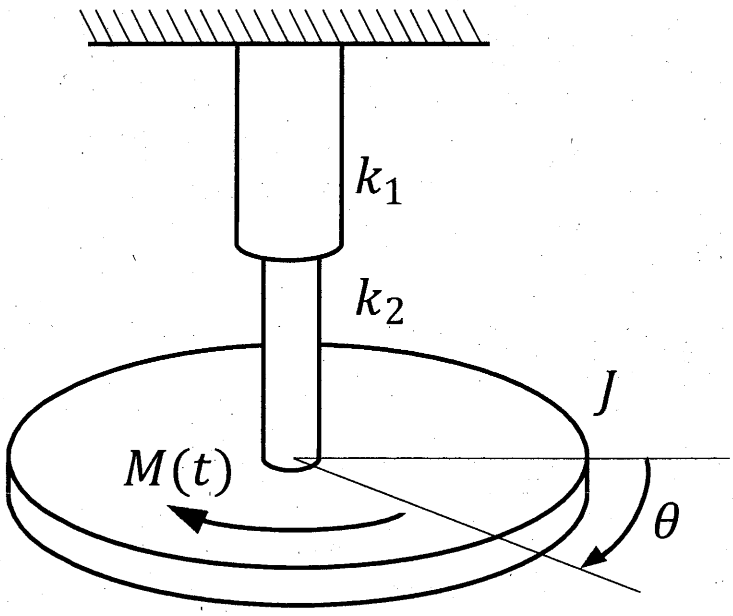


Fig. 1

2. Consider a vibration system consisting of a rigid object with mass m, length $a + b$, gravity center O, moment of inertia J about the center of gravity, and two springs with the spring constant k_1 and k_2, as shown in Fig. 2. The springs are connected to each end of the object. The object moves translationally and rotationally in the plane of the figure with sufficiently small displacements. The center of gravity O vibrates only in the vertical direction. Fig. 2-1 shows the equilibrium state of the system. The translational displacement of the center of gravity O and the angular displacement from the equilibrium position are denoted by x and θ in Fig. 2-2, respectively. Assuming that the masses of the springs are negligible, answer the following questions.

(1) Derive the equation of motion in the translational motion.
(2) Derive the equation of motion in the rotational motion.
(3) Obtain the condition that the coupling between the translational and the rotational motions does not occur.

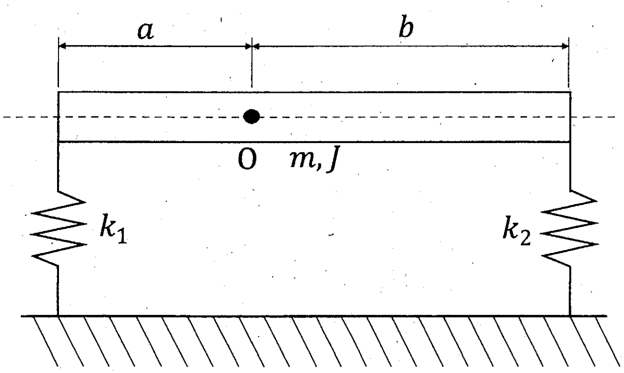


Fig. 2-1

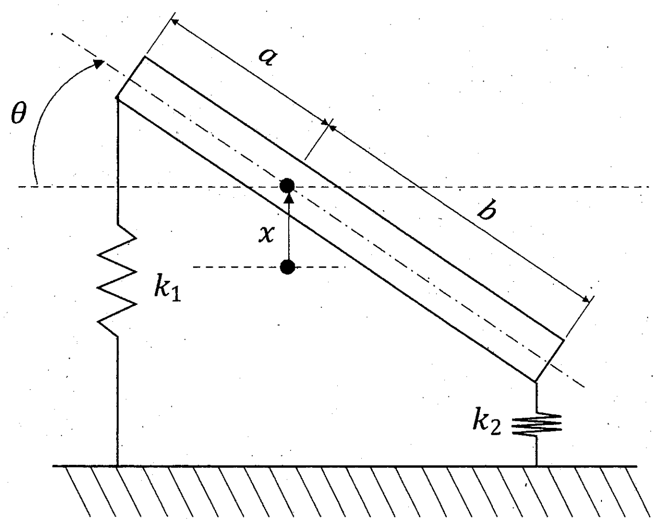


Fig. 2-2

制御工学 CONTROL ENGINEERING

1. Solve the following problems.

   (1) Consider the feedback system shown in Fig. 1. Derive the transfer functions $G_1(s)$ and $G_2(s)$ when the output $Y(s)$ is given by

   $$Y(s) = G_1(s)R(s) + G_2(s)D(s).$$

   (2) Consider a feedback system with the closed-loop transfer function $G(s)$ and the open-loop transfer function $L(s)$ as follows

   $$G(s) = \frac{L(s)}{1+L(s)}, \qquad L(s) = \frac{k}{s(s+1)(s+2)}.$$

   Find the gain margin when $k = 1$.

   (3) Let $k = 6$ for the feedback system in problem (2). Choose an appropriate explanation for the step response, and explain the reason.

   (a) The response has overshoot and converges to a constant value.
   (b) The response increases monotonically and converges to a constant value.
   (c) The response oscillates periodically.
   (d) The response diverges and goes to infinity.

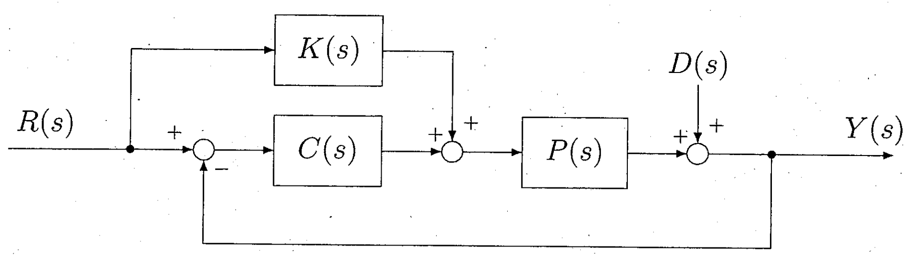


Fig. 1

2. Consider a non-linear system given by

$$\ddot{x} + \alpha\dot{x} + x^2 - x = u.$$

(1) Equilibrium points are the points where the system maintains stationary without changing in time. Find all the equilibrium points x_e of the system when $u = 0$.

(2) Let c be the largest equilibrium point x_e obtained in problem (1). Find the state equation

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u$$

of the system by linearizing the system around c, where the state vector is

$$\boldsymbol{x} = \begin{bmatrix} x - c \\ \dot{x} \end{bmatrix}.$$

(3) Suppose that a state feedback

$$u = -\boldsymbol{k}\boldsymbol{x}, \qquad \boldsymbol{k} = [11 \quad 6]$$

is applied to the system in problem (2). Then the closed-loop system has poles at -2 and p. Find the values of α and p.