英

令和５年度 秋季募集
東北大学大学院機械系４専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

令和５年８月29日(火)
Tuesday, August 29, 2023　　9:30 － 10:30（JST）

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the draft sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数学 A　MATHEMATICS A

1. Solve the following problems.

(1) Show the Taylor series of the following function $f(x)$ about $x = 1$ up to the second order

$$f(x) = x^x.$$

(2) Show the Taylor series of the following function of two variables $g(x, y)$ about $(x, y) = (1, 1)$ up to the second order

$$g(x, y) = (x + y)^2.$$

(3) Evaluate the following integral

$$\int_D y^2 \, dxdy, \quad D = \{ (x, y) \mid x^2 + y^2 \leq a^2 \},$$

where a is a positive constant.

数学 A MATHEMATICS A

2. In the Cartesian coordinate system (x, y, z), the function $f(x, y, z)$ is given by

$$f(x, y, z) = x^2 + z^2 + 2xy + 2yz.$$

$f(x, y, z)$ can be expressed as

$$f(x, y, z) = \begin{pmatrix} x & y & z \end{pmatrix} A \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

using a 3×3 symmetric matrix A. Solve the following problems.

(1) Find A.

(2) Find the eigenvalues and eigenvectors of A.

(3) Let $\boldsymbol{n}$ be an eigenvector corresponding to the smallest eigenvalue of A. The intersection of the surface $f(x, y, z) = 0$ with a plane which is perpendicular to $\boldsymbol{n}$ and does not pass through the origin is an ellipse. Find the ratio of the minor (shorter) axis to the major (longer) axis of the ellipse.

3. In the Cartesian coordinate system (x, y, z), the vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = z\,\boldsymbol{i} + x\,\boldsymbol{j} + y\,\boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors in the x, y, and z directions, respectively. The surface S is given by

$$S : z = x^2 - y^2 \qquad (x^2 + y^2 \leq a^2),$$

where a is a positive constant. Solve the following problems.

(1) Obtain $\nabla \times \boldsymbol{A}$ and $\nabla \cdot \boldsymbol{A}$.

(2) Evaluate the area of S.

(3) Evaluate the following surface integral

$$\int_S (\nabla \times \boldsymbol{A}) \cdot \boldsymbol{n}\; dS,$$

where $\boldsymbol{n}$ is the unit normal vector of S with a positive z component.

英

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試験問題冊子

数学Ｂ　MATHEMATICS B

令和５年８月29日(火)
Tuesday, August 29, 2023　　13:00 － 14:00（JST）

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, and draft sheets are provided. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

数 学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\dfrac{d^2y}{dx^2}\dfrac{d^3y}{dx^3} = \dfrac{1}{2}$

(2) $\dfrac{dy}{dx} = e^x \sin x + 2y$

2. The function $u(x,y)$ satisfies the partial differential equation

$$3\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = u + x\,.$$

When $\xi = x - 3y$ and $\eta = y$, solve the following problems.

(1) Obtain the partial differential equation for $u(\xi,\eta)$.

(2) When $z(\xi,\eta) = e^{-\eta}u$, obtain the general solution of the function $z(\xi,\eta)$ using the result of problem (1).

(3) Obtain $u(x,y)$ that satisfies $u(0,y) = e^{-2y} - 3$.

数学B MATHEMATICS B

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}[\,f(t)\,] = F(s) = \int_0^\infty f(t)\,e^{-st}\,dt\,.$$

The functions $x(t)$ and $y(t)$ satisfy the following simultaneous equations

$$\begin{aligned}\frac{dx(t)}{dt} &= -4x(t) + y(t)\,,\\ \frac{dy(t)}{dt} &= -x(t) - 4y(t)\,,\end{aligned}$$

with the initial conditions

$$x(0) = 0 \quad \text{and} \quad y(0) = 1\,.$$

Solve the following problems.

(1) Find the Laplace transforms $X(s)$ of $x(t)$ and $Y(s)$ of $y(t)$.

(2) Find $x(t)$ and $y(t)$ using the results of problem (1).

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試験問題冊子
【専門科目　Specialized Subjects】

熱力学	THERMODYNAMICS	P1~P2
流体力学	FLUID DYNAMICS	P3~P4
材料力学	STRENGTH OF MATERIALS	P5~P6
機械力学	DYNAMICS OF MECHANICAL SYSTEMS	P7~P8
制御工学	CONTROL ENGINEERING	P9~P10

令和５年８月30日（水）
Wednesday, August 30, 2023 9:30 ― 11:30（JST）

Notice

1. Do not open this test booklet until instructed to do so.

2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.

3. Select two subjects from the five subjects in the booklet and answer all the problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.

4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection by an examiner. Do not leave your seat before instructed to do so by the examiner.

熱 力 学 THERMODYNAMICS

1. Answer the following questions. Note that a thermal reservoir is a closed system with infinite heat capacity, and it supplies heat to or absorbs heat from a cycle while keeping its temperature constant. In addition, T_0, T_1, T_2, and T_3 are absolute temperatures and satisfy $T_0 > T_1 > T_2 > T_3$.

   (1) Consider a combined engine consisting of cycles A and B, as shown in Fig. 1. Cycle A operates between the thermal reservoir with temperature T_1 and that with temperature T_2. Cycle B operates between the thermal reservoir with temperature T_2 and that with temperature T_3. Cycle A receives the amount of heat Q_A from the thermal reservoir with temperature T_1. Cycle B receives the amount of heat Q_B rejected by cycle A. Each thermal efficiency of cycles A and B is assumed to be 1/2 of the Carnot efficiency determined by the respective thermal reservoir temperatures.

   a) Express the output work L_A and L_B of cycles A and B using the necessary symbols from Q_A, Q_B, T_1, T_2, and T_3, respectively.

   b) Express the temperature T_2 using T_1 and T_3 when the thermal efficiency of the combined engine is maximized.

   c) Express the temperature T_2 using T_1 and T_3 when the entropy generation of the combined engine is minimized.

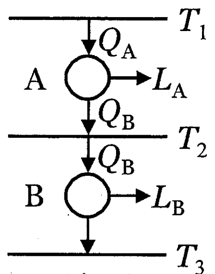

Fig. 1

   (2) Consider the cooling process of a certain mass of a substance using a reversed Carnot cycle. The initial state of this substance is a liquid phase with temperature T_1, and the final state is a solid phase with temperature T_3. This cooling process consists of the following three processes:

   Process 1: Cooling process of the liquid from temperature T_1 to T_2,

   Process 2: Phase change process from the liquid to the solid at temperature T_2, and

   Process 3: Cooling process of the solid from temperature T_2 to T_3.

   The temperature of the high-temperature thermal reservoir of the reversed Carnot cycle is assumed to be constant at T_0, and the temperature of the low-temperature thermal reservoir is assumed to be equal to that of the substance to be cooled. The heat capacities of the liquid and solid of the substance are constants C_L and C_S, respectively, and the latent heat of solidification of this mass is H. The volume change of the substance in each cooling process is negligible.

   a) Find the decrease in entropy of the cooled substance in each cooling process.

   b) Find the work input to the reversed Carnot cycle during the entire cooling process.

2. Consider a cycle in which 1 kg of an ideal gas is used as the working fluid. The cycle consists of three quasi-static processes, which are an isochoric compression process from state 1 (temperature 300 K, pressure $p_1 = 0.1$ MPa) to state 2 (pressure $p_2 = 0.4$ MPa), an isothermal expansion process from state 2 to state 3, and an isobaric cooling process from state 3 to state 1. The specific heat ratio of the working fluid is $\kappa = 1.4$, and the gas constant is $R = 0.3$ kJ/(kg·K). Answer the following questions.

(1) Draw the pressure–specific volume ($p - v$) diagram of the cycle. Indicate states 1, 2, and 3 in the diagram.

(2) Calculate the temperature at state 2.

(3) Calculate the work per cycle done to the surroundings by the cycle. Use ln 2 = 0.693, if necessary.

(4) Calculate the theoretical thermal efficiency of the cycle.

(5) Consider a process from state 2 to state 4 (pressure p_1) by adiabatic expansion. Calculate the theoretical thermal efficiency of the cycle consisting of the processes 1→2→4→1. Use $0.5^{4/7} = 0.673$, if necessary.

流 体 力 学 FLUID DYNAMICS

1. Consider a two-dimensional steady potential flow of an inviscid incompressible fluid as shown in Fig. 1. A vortex with circulation strength Γ is located at the origin in the xy plane and a source of strength m is located at the point of $z = ih$, where $z = x + iy$ is a complex variable. The complex potential $W(z)$ of this flow is given by

$$W(z) = \frac{i\Gamma}{2\pi}\log z + m\log(z - ih),$$

where h is the real number, i is the imaginary unit and $\log$ is the natural logarithm. Answer the following questions.

(1) Express the velocity potential ϕ and the stream function ψ of the flow field by defining $z = x + iy = re^{i\theta}$ and $z - ih = x + i(y - h) = r_1 e^{i\theta_1}$ using necessary symbols from π, $m, \Gamma, r, r_1, \theta$ and θ_1, where r and r_1 are the radial and θ and θ_1 are the circumferential coordinates in the polar coordinate system.

(2) Consider the velocity and the pressure at the point P ($z = h + ih$), where h is not 0.

a) Obtain the x component u_x and the y component u_y of the velocity at the point P using necessary symbols from π, m, Γ and h, respectively.

b) Obtain the pressure at the point P from Bernoulli's equation using p_0, π, ρ, m, Γ and h, where the density of the fluid is constant at ρ and the pressure at infinity is p_0.

(3) Draw the stream lines with direction arrows in the xy plane when $h = 0$.

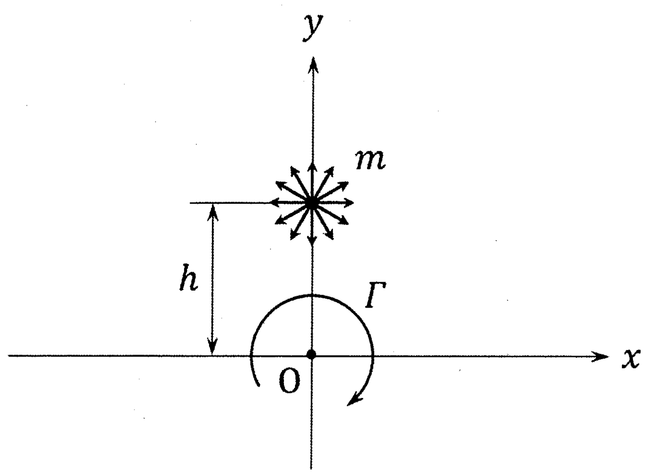

Fig. 1

流 体 力 学　FLUID DYNAMICS

2. Consider a two-dimensional steady flow of an incompressible Newtonian fluid between parallel plates at a distance h as shown in Fig. 2. A uniform flow with velocity V flows from the cross-section A. At the cross-section B at the distance L from the cross-section A, the flow is fully developed and has a parabolic velocity distribution. The pressures are p_1 and p_2 at the cross-sections A and B, respectively. The velocity distribution at the cross-section B is given by

$$u(y) = ay^2 + by + c,$$

where $a,\ b$ and c are coefficients. The density ρ and the viscosity coefficient μ of the fluid are constant and gravity is neglected. Answer the following questions.

(1) Express the coefficients $a,\ b$ and c in the velocity distribution $u(y)$ at the cross-section B using necessary symbols from h and u_{max}. The velocity u is the maximum velocity $u = u_{max}$ at $y = h/2$.

(2) Express the wall shear stress τ_w at the cross-section B using $\mu,\ h$ and u_{max}.

(3) Express the velocity u_{max} using the velocity V.

(4) Express the sum of forces F in the direction of x acting from the fluid on the upper and lower parallel plates in the section AB, using $h,\ \rho,\ V,\ p_1$ and p_2. The sum of forces F is the force acting per unit length in the direction of z.

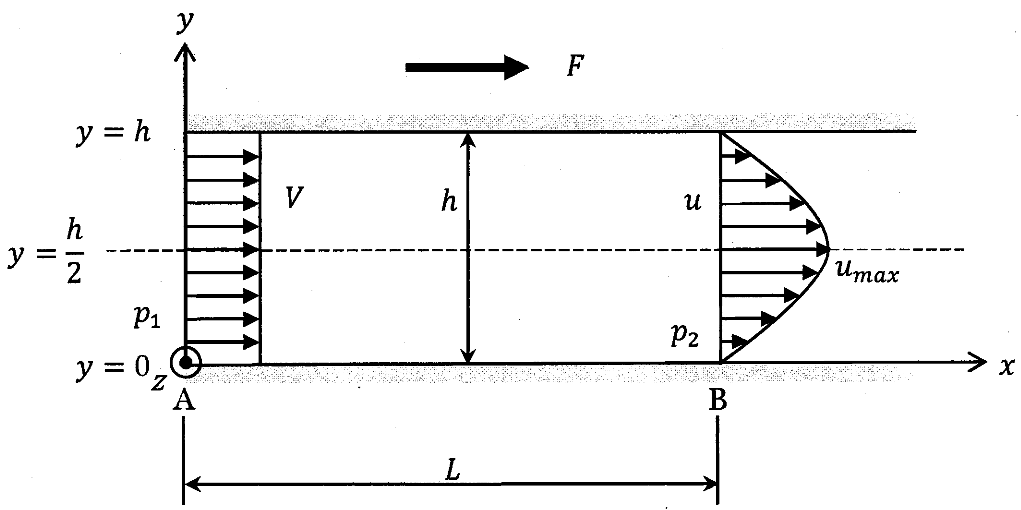

Fig. 2

材 料 力 学　STRENGTH OF MATERIALS

1. As shown in Fig. 1, consider a hollow shaft AB of outer diameter $3d$ and inner diameter $2d$, and a solid shaft CD of diameter d. The left end A of shaft AB and the right end D of shaft CD are fixed to rigid walls, respectively. There are holes in shafts AB and CD for a pin connection at position E away from the fixed end at distance L, respectively. Initially, the hole through shaft CD makes an angle β with respect to a line through two holes in shaft AB. The shear modulus of shafts AB and CD is G. Answer the following questions.

   (1) The shaft CD is twisted to make the hole positions of both shafts coincide. Determine the twisting moment M_E acting at the hole position of shaft CD.

   (2) Next, shafts AB and CD are pin-connected at position E. When M_E is released, find the twisting angle α at the pin-connected position of shaft AB. Neglect the deformation and friction of the pin and the holes.

   (3) Then, determine the maximum shear stresses τ_1 in shaft AB and τ_2 in shaft CD, respectively.

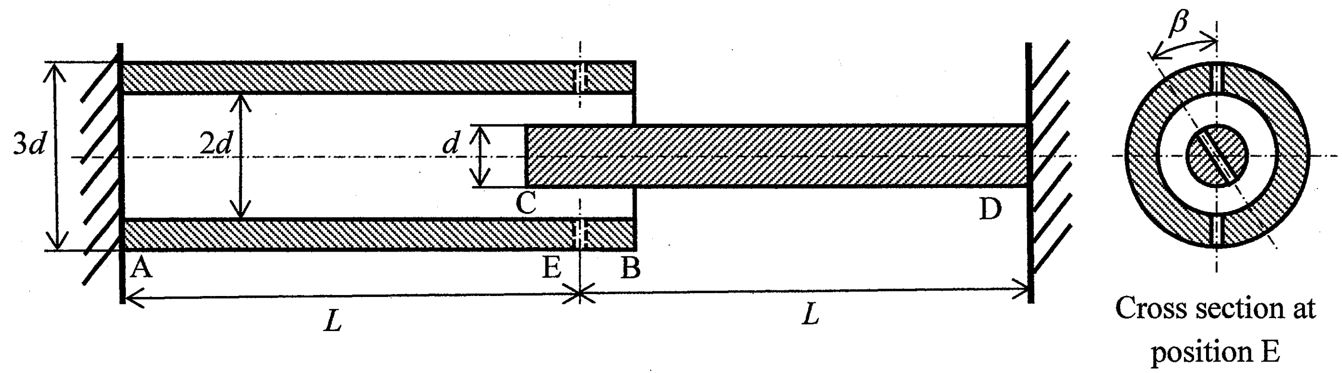

Fig. 1

2. Consider a curved beam ABC with a straight portion AB and a quarter-arc portion BC. The beam is perpendicularly fixed to a rigid wall at end A. The radius of curvature of the arc portion BC is r, and the length of the straight portion AB is $3r$. The flexural rigidity of the beam ABC is constant EI. A concentrated load P is applied downward vertically at end C. Neglect the weight of the beam. Answer the following questions.

   (1) Express the magnitude and direction of the reaction force and reactive moment at the fixed end A, respectively.
   (2) Find the vertical deflection and deflection angle at point B.
   (3) Find the bending moment at point D apart from angle θ from point B as a function of θ. Also, find the strain energy in portion BC stored by this bending moment.
   (4) Find the vertical deflection of portion BC by using Castigliano's theorem, and determine the vertical deflection at end C.

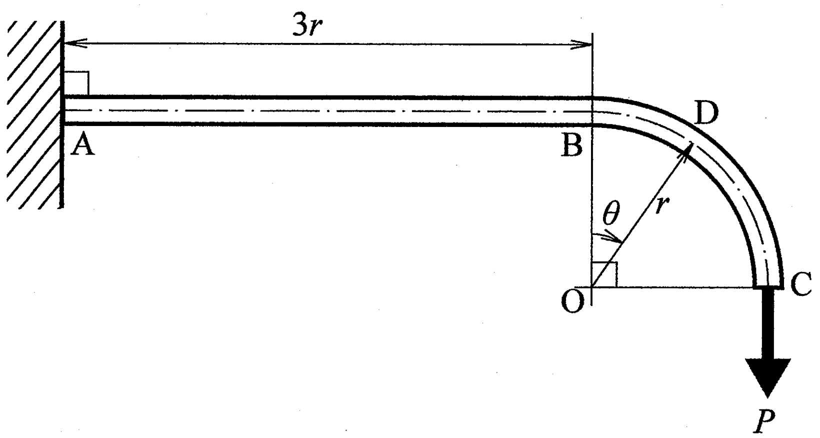

Fig. 2

機 械 力 学　DYNAMICS OF MECHANICAL SYSTEMS

1. Consider a system consisting of two springs with spring constants k_1 and k_2, a dashpot with damping coefficient c, a double pulley with a mass moment of inertia J_P, and two masses m_1 and m_2 as shown in Fig. 1. The double pulley, which consists of two wheels with radii of r_1 and r_2, is fixed in space. The ropes connecting the masses and the springs have constant lengths and do not loosen. There is no slip between the ropes and the pulley. The masses of the springs, the dashpot and the ropes are negligible. The displacement of the mass m_1 from the equilibrium position is denoted by x. Answer the following questions.

(1) Obtain the kinetic energy T of the system..
(2) Obtain the potential energy U of the system.
(3) Derive the equation of motion of the system.
(4) When the system is critically damped, determine the damping coefficient c_{cr}.

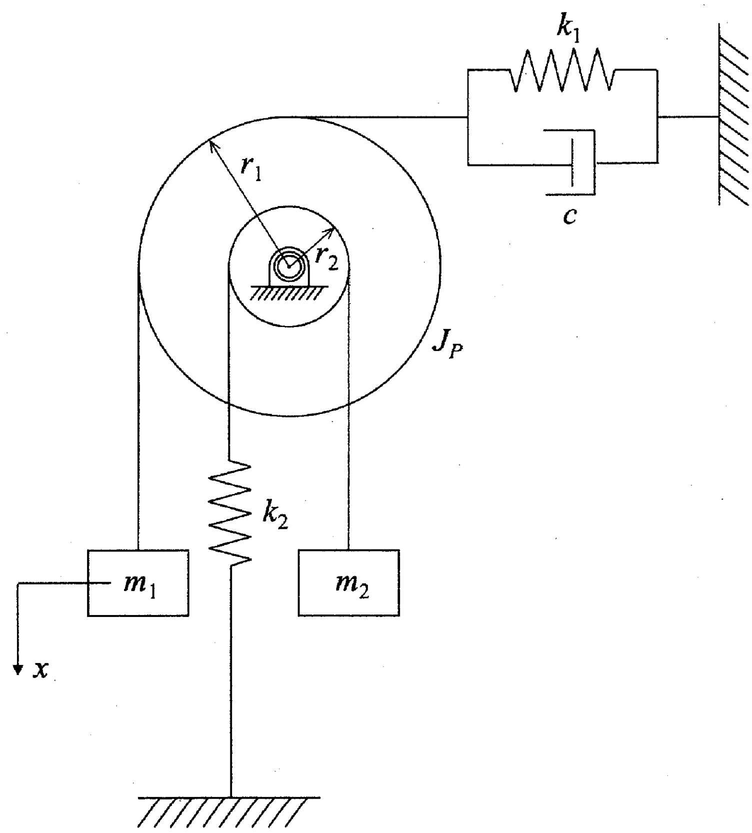

Fig. 1

2. Consider a system consisting of two pendulums with two rigid bars of length l and two masses m_1 and m_2, and three springs with spring constants k_1, k_2 and k_3, as shown in Fig. 2. The two pendulums, which are suspended from a horizontal ceiling in a gravitational field, are located vertically in the equilibrium states, respectively. The pendulums oscillate with sufficiently small amplitudes in the plane of the figure. The gravitational acceleration is g and the angular displacements of the pendulums from the equilibrium positions are denoted by θ_1 and θ_2, respectively. Assuming that the masses of the bars and the springs are negligible, answer the following questions.

(1) Derive the equations of motion of the system.

(2) When $m_1 = m$, $m_2 = 4m$, $k_1 = k_2 = k$ and $k_3 = 7k$, find the natural angular frequencies of the system.

(3) Find the amplitude ratio of θ_1 to θ_2 at each natural angular frequency obtained in question (2), and illustrate the corresponding vibration mode shapes.

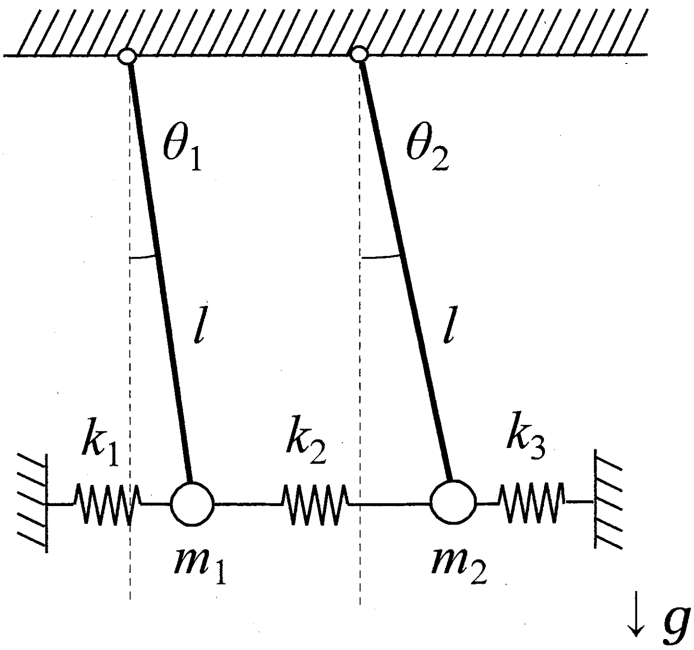

Fig. 2

制御工学 CONTROL ENGINEERING

1. Consider a feedback control system shown in Fig. 1. $P(s)$ is the transfer function of the plant. $C(s)$ and $D(s)$ are the transfer functions of the controller. $R(s)$, $Y(s)$, and $E(s)$ are Laplace transforms of reference input $r(t)$, output $y(t)$, and error $e(t)$, respectively. $P(s)$, $C(s)$ and $D(s)$ are given by

$$P(s) = \frac{4}{s(s+2)},$$

$$C(s) = K_1,$$

$$D(s) = K_2 s,$$

where K_1 and K_2 are positive constants. Solve the following problems.

(1) Obtain the closed-loop transfer function of the system $G(s)$ using $P(s)$, $C(s)$, and $D(s)$.

(2) Examine the stability of the system. Then, describe the role of $D(s)$ and the effect of $D(s)$ on the damping coefficient of the system.

(3) Find the condition of K_1 and K_2 for the steady-state velocity error is to be 0.5 or less. The steady-state velocity error is defined as the steady-state error when the unit constant velocity input $r(t) = t$ is given as the reference input.

(4) When $K_1 = 2$ and $K_2 = 1$, derive the step response of the system.

(5) When $K_2 = 0.5$, draw the root locus of the system as K_1 varies from 0 to infinity.

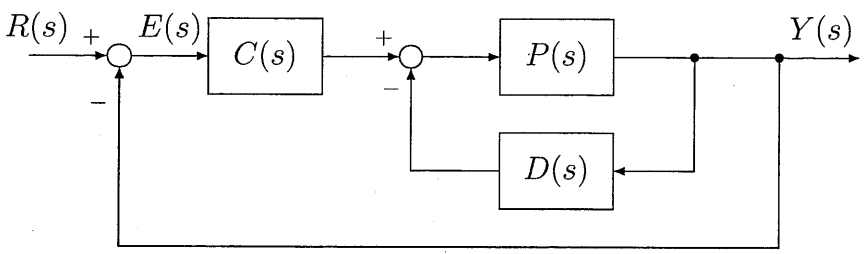

Fig. 1

制御工学 CONTROL ENGINEERING

2. Consider a dynamical system described by the following equation of motion.

$$a\frac{d^2}{dt^2}x(t) + b\frac{d}{dt}x(t) + c\sin x(t) = u(t),$$

where $a \neq 0,\ b \neq 0$, and $c \neq 0$. Solve the following problems.

(1) Find the state space equation of the system when the state variable is defined as

$$\boldsymbol{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \begin{bmatrix} x(t) \\ \frac{d}{dt}x(t) \end{bmatrix}.$$

(2) When $u(t) = 0$, show that $\boldsymbol{x}(t) = (0, 0)^T$ and $\boldsymbol{x}(t) = (\pm\pi, 0)^T$ are the points of equilibrium of this system.

(3) Linearize the state space equation obtained in problem (1) at $\boldsymbol{x}(t) = (\pi, 0)^T$.

(4) Examine the stability of the system obtained in problem (3) when $a = 1$, $b = 2$, and $c = 3$.

(5) Consider the state feedback control for the system in problem (4). Find the gains of the state feedback such that the poles of the closed-loop system are $(-3 - i, -3 + i)$, where i is the imaginary unit.