英

令和６年度実施 秋季募集
東北大学大学院機械系４専攻入学試験

試験問題冊子

数学Ａ　MATHEMATICS A

令和６年８月27日(火)
Tuesday, August 27, 2024　　9:30 － 10:30（JST）

Notice

1. Do not open this test booklet until instructed to do so.
2. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the draft sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

数学 A　MATHEMATICS A

1. Solve the following problems.

   (1) Evaluate the following limit

   $$\lim_{x \to 0} \frac{e^x - \cos x}{2 \tan x}.$$

   (2) Evaluate the following integral

   $$\int_0^{\infty} x e^{-2x}\, dx.$$

   (3) Find the extremums of $x - 2y$ subjected to the constraint $x^2 + 2y^2 - 1 = 0$.

数学 A　MATHEMATICS A

2. The matrix A is given by

$$A = \begin{pmatrix} a & 1-a \\ 1+a & -a \end{pmatrix},$$

where a is a constant. Solve the following problems, where n is a positive integer.

(1) Find the eigenvalues and eigenvectors of A.

(2) Obtain A^n.

(3) Find the rank of the following 4×4 matrix

$$\begin{pmatrix} A^{2n} & A^{2n+1} \\ A^{2n+1} & A^{2n} \end{pmatrix}.$$

数学A　MATHEMATICS A

3. In the Cartesian coordinate system (x, y, z), the surface S is given by

$$S : x^2 + y^2 + z^2 = 1, \quad x^2 + y^2 \leq z^2 \tan^2 \alpha, \quad z \geq 0,$$

where $0 < \alpha < \pi/2$. The unit normal vector of S with a positive z component is denoted by $\boldsymbol{n}$. Solve the following problems.

(1) Obtain $\boldsymbol{n}$.

(2) Find α when the area of S is π.

(3) Consider the case $\alpha = \pi/4$. The vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = (x + y)\, z^2\, \boldsymbol{i} + yz^2\, \boldsymbol{j} + xz^2\, \boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors in the x, y, and z directions, respectively. Evaluate the following surface integral

$$\int_S (\nabla \times \boldsymbol{A}) \cdot \boldsymbol{n}\, dS.$$

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東北大学大学院機械系４専攻入学試験

試験問題冊子

数学Ｂ　MATHEMATICS B

令和６年８月27日(火)
Tuesday, August 27, 2024　13:00 － 14:00（JST）

Notice

1. Do not open this test booklet until instructed to do so.
2. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

数学B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $(x-a)\dfrac{dy}{dx}-(x-a)^3-3y=0$　　(a is a constant)

(2) $y\dfrac{d^2y}{dx^2}+\left(\dfrac{dy}{dx}\right)^2-2y^2=-x$

数学B　MATHEMATICS B

2. The Fourier transform $F(\omega)$ of a function $f(x)$ and its inverse transform are defined by

$$F(\omega) = \int_{-\infty}^{\infty} f(x)\, e^{-i\omega x}\, dx\,,$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, e^{i\omega x}\, d\omega\,.$$

Solve the following problems when $f(x) = e^{-ax^2}$. Here, a is a constant satisfying $a > 0$.

(1) Derive

$$\frac{dF(\omega)}{d\omega} = -\frac{\omega}{2a} F(\omega)\,.$$

(2) Find $F(\omega)$ using the result of problem (1). If necessary, use $F(0) = \sqrt{\dfrac{\pi}{a}}$.

(3) A function $g(x)$ satisfies the following equation

$$\int_{-\infty}^{\infty} g(y)\, g(x-y)\, dy = e^{-\frac{x^2}{2}}\,.$$

Obtain $g(x)$.

数学B　MATHEMATICS B

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}\,[\,f(t)\,] = F(s) = \int_0^{\infty} f(t)\,e^{-st}\,dt\,.$$

Solve the following problems.

(1) Obtain the Laplace transform of $\dfrac{1}{\sqrt{t}}$ by using $\mathcal{L}\left[\sqrt{t}\right] = \dfrac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$.

(2) A function $g(t)$ is given by $g(t) = \displaystyle\int_{-\infty}^{\infty} t\,e^{-tx^2}\,dx\,.$
By using $G(s) = \mathcal{L}\,[\,g(t)\,] = \dfrac{\pi}{2s^{\frac{3}{2}}}$, derive the following relation

$$\frac{dg(t)}{dt} = \frac{\sqrt{\pi}}{2}\frac{1}{\sqrt{t}}\,.$$

(3) Evaluate $\displaystyle\int_{-\infty}^{\infty} e^{-x^2}\,dx$ by using $\mathcal{L}\left[\sqrt{t}\right]$ and $G(s)$ in problem (2).

(4) Evaluate $\displaystyle\int_{-\infty}^{\infty} x^2\,e^{-x^2}\,dx\,.$

英

令和 6 年度実施 秋季募集
東北大学大学院機械系 4 専攻入学試験

試験問題冊子
【専門科目　Specialized Subjects】

令和 6 年 8 月 28 日（水）
Wednesday, August 28, 2024　9:30 － 11:30（JST）

Notice

1. Do not open this test booklet until instructed to do so.
2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.
3. Select two subjects from the five subjects in the booklet and answer all the problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

熱 力 学 THERMODYNAMICS

1. As shown in Fig. 1, vessels A and B are connected via a tube and a valve. Vessel B has an adiabatic piston that moves smoothly in the horizontal direction. The two vessels, the connecting tube and the valve are surrounded by an adiabatic wall. The volume of the connecting tube is negligible. In state 1, the piston is fixed where the volumes of vessels A and B are equal, and an ideal gas with specific heat ratio κ is filled into vessel A at pressure p_1, temperature T_1 and specific volume v_1. The valve is closed, and the inside of vessel B is a vacuum. Answer the following questions.

(1) From state 1, when the valve is opened and sufficient time passes, the gases in vessels A and B are in equilibrium at temperature T_2 and pressure p_2. This is state 2. Express the temperature T_2, the pressure p_2 and the increase in specific entropy Δs_{12} during the change from state 1 to state 2 using the necessary symbols of p_1, T_1, v_1 and κ, respectively.

(2) From state 2, the piston is unfixed, and the gas is adiabatically expanded in a quasi-static process until it reaches temperature T_3. This is state 3. Express the pressure p_3 and specific volume v_3 at state 3 using the necessary symbols of p_1, T_1, T_3, v_1 and κ, respectively.

(3) From state 1, when the valve is opened and then closed, the gas remaining in vessel A changes reversibly and adiabatically; the gas pressures in vessels A and B are equal to p_4, respectively, while the temperatures of the gas in vessels A and B are different, T_{A4} and T_{B4}, respectively. Express the pressure p_4 and temperature T_{A4} using the necessary symbols of p_1, T_1, v_1 and κ, respectively.

(4) Show the relation of magnitudes among T_1, T_2, T_3, T_{A4} and T_{B4} in descending order using equal and greater-than signs when the specific volume of state 3 is $v_3 = 4\, v_1$.

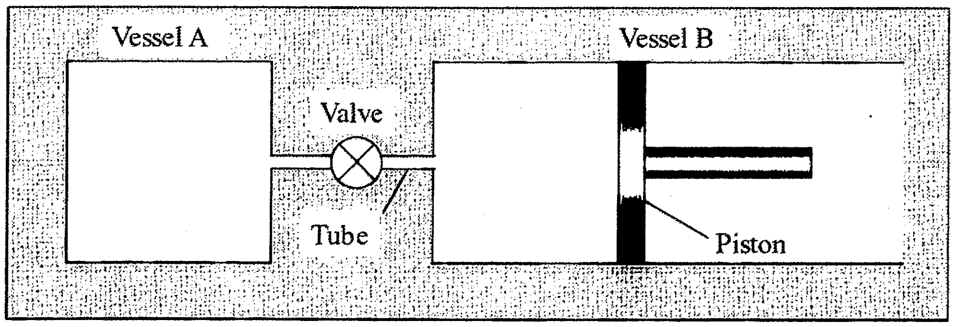

Fig. 1

2. Consider the vapor-liquid equilibrium of a certain pure substance. Figures 2 and 3 show the pressure-temperature ($p-T$) and pressure-volume ($p-v$) diagrams of this pure substance, respectively. The state of the pure substance is changed from state C to state D, keeping the temperature at 100 K constant. The thermodynamic properties of the pure substance in its saturated state at 100 K are shown in Table 1. Answer the following questions.

(1) Show the specific Gibbs free energy g using temperature T, specific enthalpy h and specific entropy s.

(2) Show that the specific Gibbs free energy of the system does not change from state C_S to state D_S in Fig. 3.

(3) Calculate the value of specific entropy s_G in Table 1.

(4) Show the gradient of curve AB at point S, which is on line CD in Fig. 2, using temperature T, the specific volume of the liquid phase v_L, the specific volume of the gas phase v_G and the latent heat of vaporization r.

(5) Using the values in Table 1, calculate the saturation pressure at a temperature of 101 K with three significant figures. Assume that the curve AB in the vicinity of point S can be considered as a straight line in part.

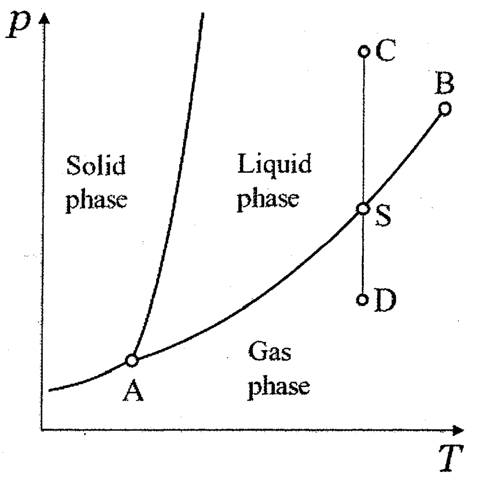

Fig. 2

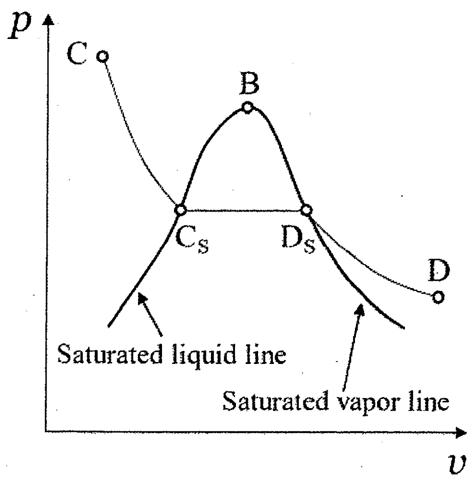

Fig. 3

Table 1

	Saturated liquid	Saturated vapor
Pressure [MPa]	0.100	
Density [kg/m^3]	1000.0	5.000
Enthalpy [kJ/kg]	−100.0	150.0
Entropy [kJ/(kg·K)]	3.000	s_G

流 体 力 学　FLUID DYNAMICS

1. As shown in Fig. 1, a container filled with water is fixed on a horizontal floor. There is a hole of diameter D at depth h_1. A circular pipe of diameter D and length L is horizontally connected to the container at depth h_2. Water is ejected into the atmosphere in the same horizontal direction at constant velocities U_1 and U_2 from the hole and the pipe outlet, respectively. The water level is assumed to be constant. The gravitational acceleration is g. The viscosity μ and density ρ of water are constant. The pipe friction loss ΔP is taken into account, while other pressure losses can be neglected. ΔP is given by using the pipe friction coefficient λ as

$$\Delta P = \lambda \frac{L}{2D} \rho U_2^2 \ .$$

Answer the following questions.

(1) Express the velocity U_1 using h_1 and g.
(2) Express the Reynolds number *Re* of the flow in the circular pipe using μ, ρ, D and U_2.
(3) Express the wall shear stress τ acting on the circular pipe using λ, ρ and U_2.
(4) Express the magnitude of force F exerted on the container due to the water ejections using ρ, D, U_1 and U_2.
(5) Express the depth ratio h_2/h_1 using λ, D and L, when U_2 and U_1 are equal.

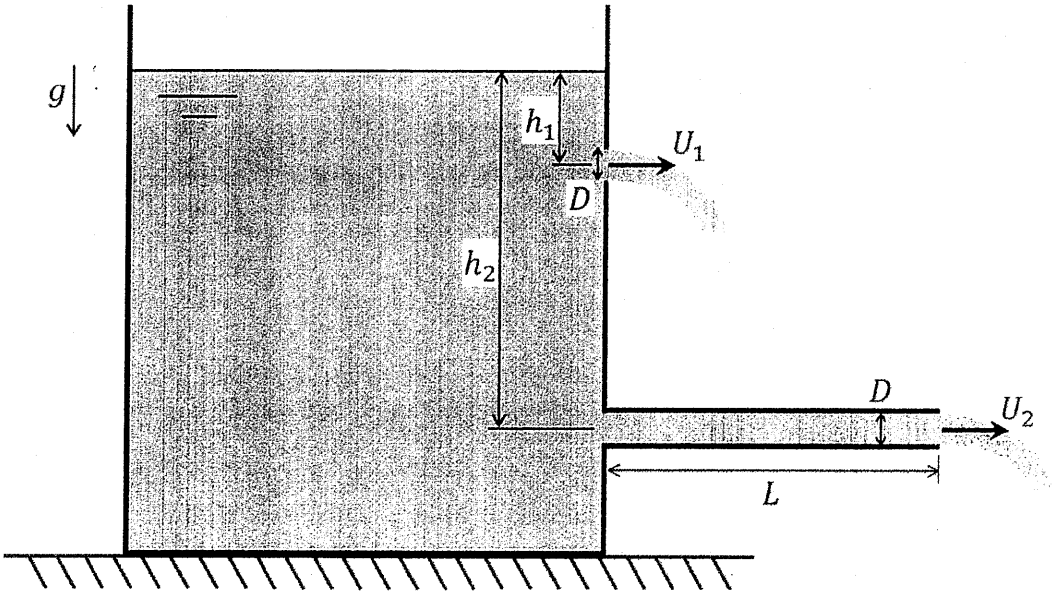

Fig. 1

2. Consider a two-dimensional steady potential flow of an inviscid incompressible fluid. The complex potential $W(z)$ of the flow is given as follows:

$$W(z) = Cz^3.$$

The complex variable z is given by $z = re^{i\theta}$ in the polar form, where r and θ are the radial and angular coordinates, respectively. i is the imaginary unit, C is a positive real number, and $0 \leq \theta \leq \pi/2$. Answer the following questions.

(1) Express the velocity potential ϕ and the stream function ψ of the flow field as a function of r and θ, respectively.

(2) Obtain the radial velocity V_r and the circumferential velocity V_θ.

(3) Show that the vorticity of the flow field is 0.

(4) Obtain the pressure $p(r)$ at $\theta = 0$, with the pressure at the origin as p_0 and the density of the fluid as ρ.

(5) Obtain θ_1 and θ_2 where the stream function ψ is 0. Draw the streamlines and the flow directions in the range of $\theta_1 \leq \theta \leq \theta_2$.

1. As shown in Fig. 1, a rigid bar ABCD of length $3L$ is pinned to bars BE and BF at point B and to a bar DG at point D, respectively, so that the rigid bar is horizontal. The bars BE and DG are pinned to rigid ceilings, and the bar BF is pinned to a rigid floor. The lengths of sections AB and BD of the rigid bar ABCD are L and $2L$, respectively. Young's modulus and the cross-sectional area of the bars BE, BF, and DG are E and S, respectively. The angles of the bars BE, BF, and DG from the rigid bar ABCD are 30°, 60°, and 90°, respectively. Vertical concentrated loads W and $2W$ are applied downward at point A and the middle point C of the section BD, respectively. Neglect the weight of the rigid bar ABCD and the bars BE, BF, and DG. Answer the following questions.

(1) Determine the axial forces in the bars BE, BF, and DG.
(2) Determine the axial deformation of the bars BE, BF, and DG.
(3) Determine the vertical displacement of point B.
(4) Determine the tilt angle of the rigid bar ABCD.

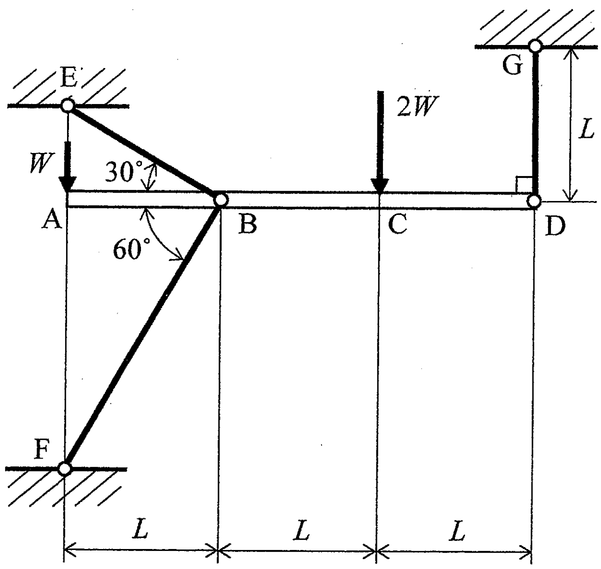

Fig. 1

2. Answer the following questions.

(1) As shown in Fig. 2(a), a beam AB of length $2L$ is fixed to a rigid wall at left end A. The flexural rigidity of the beam AB is EI. A moment M_0 and a concentrated load W are applied at right end B. Neglect the weight of the beam. Determine the deflection angle at right end B.

(2) As shown in Fig. 2(b), a stepped beam ABCDE of length $4L$ is fixed to rigid walls at left end A and right end E. The lengths of the sections AB, BD, and DE are L, $2L$, and L, respectively. The flexural rigidities of the sections AB, BD, and DE are $2EI$, EI, and $2EI$, respectively. A concentrated load W is applied at the middle point C of the section BD. Neglect the weight of the beam. Determine the deflection at the middle point C.

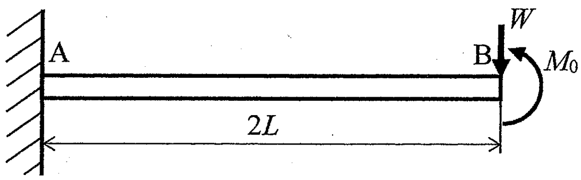

Fig. 2(a)

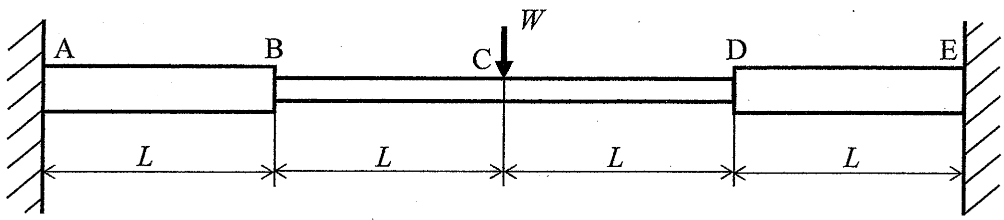

Fig. 2(b)

機械力学 DYNAMICS OF MECHANICAL SYSTEMS

1. Consider a system which consists of a mass m, a spring with spring constant k and a dashpot with damping coefficient c, as shown in Fig. 1. The mass m is connected to a wheel by the spring and the dashpot, and vibrates only in the vertical direction. The displacement of mass m from the equilibrium position is denoted by x. The wheel does not leave the road, and the wheel moves at velocity v in the horizontal direction on a sinusoidal road with amplitude a and wavelength ℓ. The wheel is at the highest position of the road when time $t = 0$. Assume that the wheel is sufficiently small, and the masses of the wheel, spring and the dashpot are negligible. When the system is in a steady-state, answer the following questions.

   (1) Derive the equation of motion of the system.

   (2) Obtain the equation showing the relationship between amplitude A of the mass m and the velocity v.

   (3) Assuming $c = 0$, determine the critical velocity v_{cr} of the system.

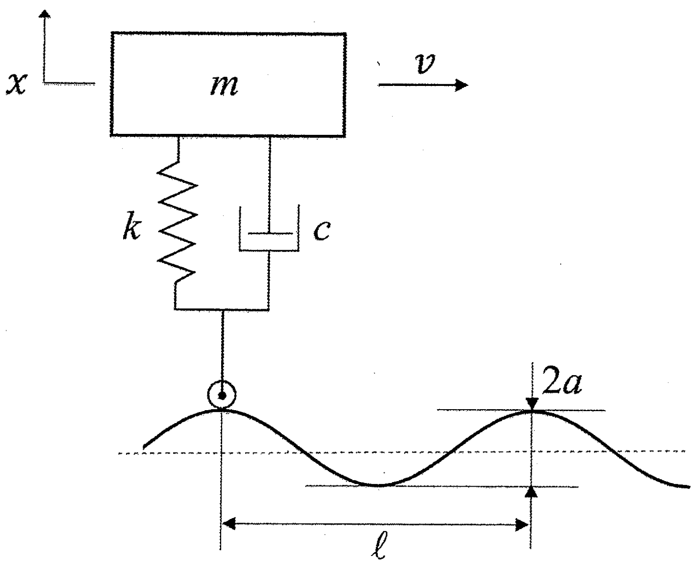

Fig. 1

2. Consider a vibration system composed of a rigid bar of length 4ℓ, two masses m fixed at both ends of the rigid bar, and two springs with spring constants k_1 and k_2, as shown in Fig. 2. One end of each spring is connected to the rigid bar at distance ℓ from the gravitational center G, and the other end is connected to a fixed wall. The mass of the rigid bar is negligible. The rigid bar is kept vertical and the springs are kept horizontal in the equilibrium state. Denoting the horizontal displacement of G by x and the rotational angle of the rigid bar by θ, and assuming that θ is sufficiently small, answer the following questions.

(1) Obtain the moment of inertia I around the gravitational center G.
(2) Represent the displacement of each spring using x and θ.
(3) Derive the equations of motion of the system.
(4) Obtain the natural angular frequencies of the system.
(5) When $k_1 = k_2 = k$, explain how the system vibrates in the first and second modes, respectively.

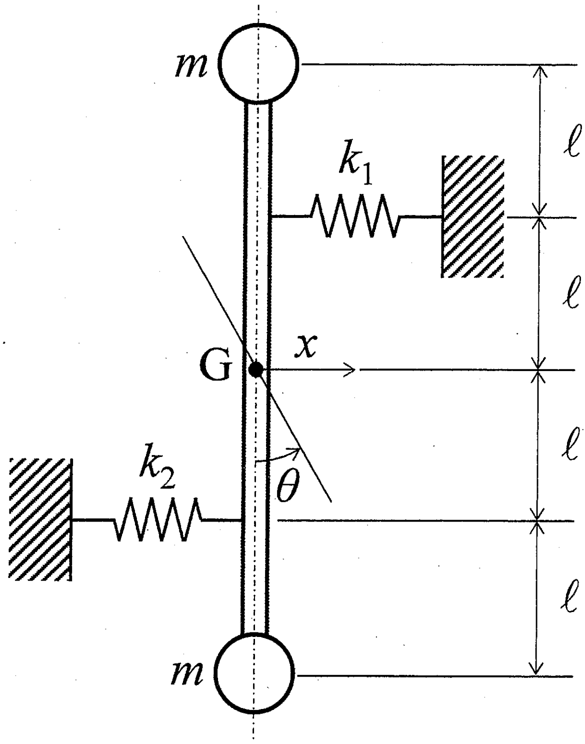

Fig. 2

1. Consider the feedback system shown in Fig. 1. The transfer functions $C(s)$ and $P(s)$ are given by

$$C(s) = k \quad (k > 0), \quad P(s) = \frac{2-s}{(s+1)(s+2)}.$$

Solve the following problems.

(1) Derive the time response when a unit step function is applied to $P(s)$. Also, draw its outline.

(2) Draw the vector locus of $P(s)$, and derive the gain margin of $P(s)$.

(3) When the output is expressed by $Y(s) = G_r(s)R(s) + G_d(s)D(s)$, describe $G_r(s)$ and $G_d(s)$ using $P(s)$, $Q(s)$, and $C(s)$.

(4) Suppose $Q(s) = 0$. Obtain the range of k for this feedback system to be stable.

(5) Suppose $Q(s) = \dfrac{s}{(s+1)(s+2)}$ and $D(s) = 0$. Derive the steady-state error $e(\infty)$ when a unit step function is applied to the feedback system.

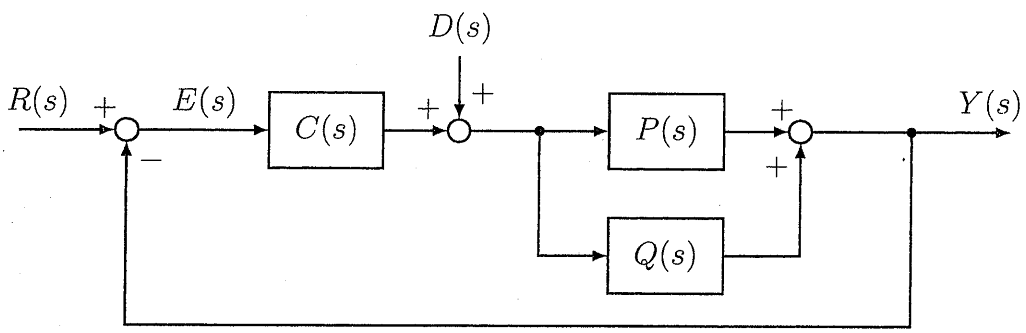

Fig. 1

制御工学 CONTROL ENGINEERING

2. Consider the feedback control system expressed by the following equation

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u,$$

$$u = -kx_1 - kx_2.$$

Solve the following problems.

(1) When this system is expressed in the following form

$$\dot{\boldsymbol{x}} = \boldsymbol{H}\boldsymbol{x}, \quad \boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix},$$

find the elements of the matrix $\boldsymbol{H}$.

(2) Find the range of k for this control system to be stable.

(3) Define the evaluation function of this control system by

$$J = \int_0^\infty (\boldsymbol{x}^T\boldsymbol{x} + \lambda u^2)\, dt = \int_0^\infty \boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x}\, dt.$$

Then, a positive definite matrix $\boldsymbol{P}$ exists for which the following relation holds

$$J = \boldsymbol{x}^T(0)\boldsymbol{P}\boldsymbol{x}(0).$$

Derive the following equation

$$\boldsymbol{H}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{H} = -\boldsymbol{Q}.$$

(4) Find the elements of the matrix $\boldsymbol{P}$ using λ and k from the equation given in problem (3).

(5) Find the feedback gain k that minimizes the evaluation function when the initial value is $\boldsymbol{x}(0) = [1, 0]^T$.

(6) When $\lambda = 1$, find $x_1(t)$ with the initial value given in problem (5).

(7) When $\lambda = 0.01$, state how $x_1(t)$ changes compared to the response obtained in problem (6).