英

令和 7 年度(2025 年度)実施 秋季募集
東北大学大学院機械系 4 専攻入学試験

試験問題冊子

数学A　MATHEMATICS A

令和 7 年(2025 年)8 月 26 日(火)
Tuesday, August 26, 2025　9:30 － 10:30 (JST)

Notice

1. Do not open this test booklet until instructed to do so.
2. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the draft sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

数学 A　MATHEMATICS A

1. Solve the following problems.

(1) Evaluate the following limit

$$\lim_{x \to 0} \frac{\tan x - x}{x^3}.$$

(2) Evaluate the following multiple integral

$$\iiint_D \log \sqrt{x^2 + y^2 + z^2}\, dxdydz,$$

where the region D is given by

$$D = \{\ (x, y, z) \mid x^2 + y^2 + z^2 \leq 1\ \}.$$

(3) Evaluate the following integral

$$\int \frac{x^n}{x-1}\, dx,$$

where n is a positive integer. Use the summation symbol if necessary.

数 学 A　MATHEMATICS A

2. The matrix A is given by

$$A = \begin{pmatrix} 3 & 1 & -1 \\ -1 & 5 & 1 \\ -2 & 2 & 4 \end{pmatrix}.$$

Solve the following problems.

(1) Find the eigenvalues of A.

(2) Find the eigenvectors of A.

(3) Find the Jordan normal form of A.

(4) Obtain A^n, where n is a positive integer.

数学A　MATHEMATICS A

3. In the Cartesian coordinate system (x, y, z), the vector field $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = xz\,\boldsymbol{i} + yz\,\boldsymbol{j} + z^2\,\boldsymbol{k},$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, and $\boldsymbol{k}$ are the fundamental vectors in the x, y, and z directions, respectively. The region V is given by

$$V : x^2 + y^2 - z^2 \leq 1, \quad 0 \leq z \leq 1.$$

Solve the following problems.

(1) Obtain $\nabla \times \boldsymbol{A}$ and $\nabla \cdot \boldsymbol{A}$.

(2) The cylindrical coordinates (r, θ, z) are defined as

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = z,$$

where $0 \leq \theta \leq 2\pi$. Express V using the cylindrical coordinates.

(3) Evaluate the following surface integral

$$\int_S \boldsymbol{A} \cdot \boldsymbol{n}\; dS,$$

where S is the surface of V and $\boldsymbol{n}$ is the unit outward normal vector of S.

英

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試験問題冊子

数学B　MATHEMATICS B

令和7年(2025年)8月26日（火）
Tuesday, August 26, 2025　13:00 － 14:00（JST）

Notice

1. Do not open this test booklet until instructed to do so.
2. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

数学 B　MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\dfrac{dy}{dx} = \dfrac{2y^2 - 6xy + x^2}{2xy - 6x^2}$

(2) $x^2\dfrac{d^2y}{dx^2} - 2x\dfrac{dy}{dx} + 2y = x \qquad (x > 0)$

数学B　MATHEMATICS B

2. The function $u(x,t)$ satisfies the partial differential equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad (0 < x < 1,\ t > 0)\,.$$

Solve the following problems.

(1) Obtain the ordinary differential equations for $X(x)$ and $T(t)$, respectively, when $u(x,t) = X(x)T(t)$.

(2) Obtain the general solutions of $X(x)$ and $T(t)$ in problem (1).

(3) Obtain the general solution of $u(x,t)$ that satisfies the boundary conditions

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = 0\,; \qquad \left.\frac{\partial u}{\partial x}\right|_{x=1} = 0\,.$$

(4) Obtain $u(x,t)$ that satisfies the boundary conditions of problem (3) when $u(x,0) = \sin^2 \pi x$.

数学B　MATHEMATICS B

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}\,[\,f(t)\,] = F(s) = \int_0^\infty f(t)\,e^{-st}\,dt\,.$$

The function $g(t)$ is given by

$$g(t) = \sum_{n=0}^{\infty} h(t-nT)\,,$$

where T is a positive constant and

$$h(t) = \begin{cases} 1 & \left(0 \leq t < \dfrac{T}{2}\right) \\ 0 & \left(t < 0\,,\ t \geq \dfrac{T}{2}\right) \end{cases}.$$

Solve the following problems.

(1) Obtain the Laplace transform of $h(t)$.

(2) Obtain the Laplace transform of $g(t)$.

(3) Evaluate $\displaystyle\int_0^\infty \left\{ \int_0^t g(u)\,du \right\} e^{-2t}\,dt\,.$

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東北大学大学院機械系４専攻入学試験

試験問題冊子
【専門科目　Specialized Subjects】

熱力学	THERMODYNAMICS	P1~P2
流体力学	FLUID DYNAMICS	P3~P4
材料力学	STRENGTH OF MATERIALS	P5~P6
機械力学	DYNAMICS OF MECHANICAL SYSTEMS	P7~P8
制御工学	CONTROL ENGINEERING	P9~P10

令和７年(2025 年)8 月 27 日（水）
Wednesday, August 27, 2025　　9:30 – 11:30（JST）

Notice

1. Do not open this test booklet until instructed to do so.

2. A test booklet, answer sheets, draft sheets, and two selected-subjects forms are provided. Put your examinee number on each of the answer sheets, the draft sheets, and the form.

3. Select two subjects from the five subjects in the booklet and answer all the problems in each subject. Indicate your selection on the selected-subjects form. Use one set of two answer sheets for each subject, and use one sheet per problem.

4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the other sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

熱 力 学　THERMODYNAMICS

1. Consider a quasistatic process in which a gas with pressure p and specific volume v is adiabatically expanded from state 1 to state 2. Pressure at state 1 is p_1 and specific volume at state 1 is v_1. Specific volume at state 2 is v_2. In this process, pv^n remains constant ($n > 1$). Answer the following questions.

(1) Explain the physical meanings of a quasistatic process and an adiabatic process.

(2) Explain real gas effects which are considered in the van der Waals equation of state in comparison with the equation of state of an ideal gas.

(3) Consider the case where the gas is a real gas. Here, specific internal energy at state 1 is u_1 and that at state 2 is u_2.

a) Express the change in specific internal energy, $u_2 - u_1$, using p_1, v_1, v_2 and n.

b) Show that the change in specific enthalpy, $h_2 - h_1$, equals $n\ (u_2 - u_1)$. Here, specific enthalpy at state 1 is h_1 and that at state 2 is h_2.

(4) Consider the case where the gas is an ideal gas with the specific heat ratio κ and gas constant R.

a) Show that pv^κ remains constant during this process.

b) When $v_2 = 2\,v_1$, express the temperature at state 2 and the work done by the gas during this process using necessary symbols from p_1, v_1, κ and R, respectively.

2. Figure 1 is the temperature-pressure ($T-p$) diagram showing the changes in temperature T and pressure p during isenthalpic processes of a real gas with a specific enthalpy h. Dashed line A is the line connecting the local maximal points of isenthalpic lines. Point B is the intersection point between dashed line A and the line of $p = 0$. c_p and v are specific heat at constant temperature and specific volume, respectively. Answer the following questions.

(1) Give the names of dashed line A and point B, respectively.

(2) Show the total differential of specific entropy s when s is a function of T and p.

(3) Show the differential change in specific enthalpy dh can be expressed by the following equation.

$$dh = c_p dT + \left\{ v - T\left(\frac{\partial v}{\partial T}\right)_p \right\} dp$$

(4) Using c_p, T, v and $\left(\frac{\partial v}{\partial T}\right)_p$, show $\left(\frac{\partial T}{\partial p}\right)_h$ for the isenthalpic process.

(5) Table 1 shows the properties of a real gas at pressure of 1 MPa. Explain whether this real gas can be cooled by throttling expansion at the state of pressure of 1 MPa and temperature of 500 K and give the reason. Include in the answer whether this state is located on the left or right side of dashed line A in Fig. 1. Assume that the isenthalpic line in the temperature range of Table 1 can be considered as a straight line locally.

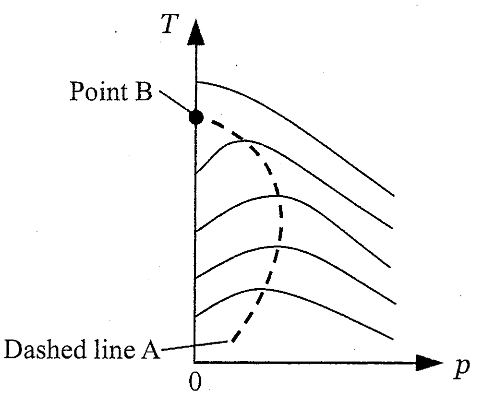


Fig. 1

Table 1

Temperature T [K]	Specific volume v [m^3/kg]	Specific enthalpy h [kJ/kg]	Specific entropy s [kJ/(kg·K)]
495	2.270	2916	7.898
500	2.295	2926	7.918
505	2.320	2936	7.938

Note: Pressure is 1 MPa.

流 体 力 学　FLUID DYNAMICS

1. As shown in Fig. 1, a cone is rotating within parallel flat plates. There is an incompressible Newtonian fluid with the constant viscosity μ and the constant density ρ between the plates that flows in a fully developed steady laminar state by the cone rotation. The tip of cone is attached at the origin O of a cylindrical coordinate with a rotational z-axis perpendicular to the plates, a radial r-direction, and a circumferential θ-direction. The radius R, the height H and the rotational angular velocity Ω of the cone, the angle between the cone and lower plate α, and the distance between the cone and upper plate δ are constant. Assuming that the angle α is sufficiently small and a centrifugal force can be ignored, the radial velocity v_r of the fluid is assumed to be zero. The circumferential velocity v_θ of the fluid can be expressed by the following equation.

$$v_\theta(r,z) = \begin{cases} \dfrac{\Omega}{\tan\alpha} z & (0 \le r \le R, \quad 0 \le z < H) \\ \dfrac{r\Omega}{\delta}(H+\delta-z) & (0 \le r \le R, \quad H \le z \le H+\delta) \end{cases}$$

A wall shear stress on the plates τ_{wall} can be expressed by the following equation.

$$\tau_{\mathrm{wall}} = \mu \frac{\partial v_\theta}{\partial z} \quad (z = 0,\ H+\delta)$$

Answer the following questions.

(1) When the characteristic length is R, and the characteristic velocity is $R\Omega$, express the Reynolds number Re of the flow using ρ, μ, Ω and R.

(2) Express the averaged value of the magnitude of circumferential velocity $\overline{|v_\theta|}$ of the fluid in the region between the cone and the upper plate ($0 \le r \le R$, $H \le z \le H+\delta$) using Ω and R.

(3) Express the magnitude of the torque $|T_{\mathrm{up}}|$ acting on the region of the upper plate, using μ, δ, Ω and R.

(4) Express the magnitude of the torque $|T_{\mathrm{low}}|$ acting on the region of the lower plate ($0 \le r \le R$, $z = 0$) using α, μ, Ω and R.

(5) Express δ using H in the case that $|T_{\mathrm{up}}|$ and $|T_{\mathrm{low}}|$ are equal.

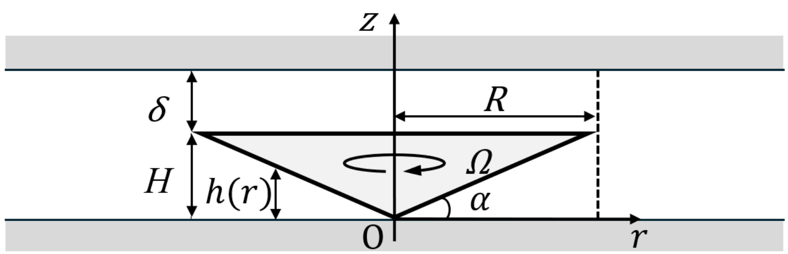


Fig. 1

流 体 力 学 FLUID DYNAMICS

2. Consider a steady two-dimensional potential flow of an inviscid and incompressible fluid. The complex potential $W(z)$ of the flow is given as follows:

$$W(z) = i\,U\,z + m \log z,$$

where U and m are positive real numbers, log is the natural logarithm and $i = \sqrt{-1}$. The complex variable z is given by $z = re^{i\theta}$ in the polar form. The r and θ are the radial and circumferential coordinates, and the range of θ is $-\pi < \theta \leq \pi$. Answer the following questions.

(1) Express the velocity potential $\phi(r, \theta)$ and the stream function $\psi(r, \theta)$ using necessary symbols from U, m, r and θ, respectively.

(2) Express the radial velocity v_r and the circumferential velocity v_θ using necessary symbols from U, m, r and θ, respectively.

(3) Obtain the radial velocity v_r and the circumferential velocity v_θ at infinity with $\theta = -\pi/2$, 0, $\pi/2$ and π, respectively.

(4) Obtain the coordinates (r, θ) of the stagnation point.

(5) Obtain the coordinates (r, θ) of the points where the streamlines passing through the stagnation point intersect the lines $\theta = 0$ and $\theta = \pi$, respectively, excepting the origin.

(6) Choose one figure of the streamlines of the flow field from (a) to (h) in Fig. 2.

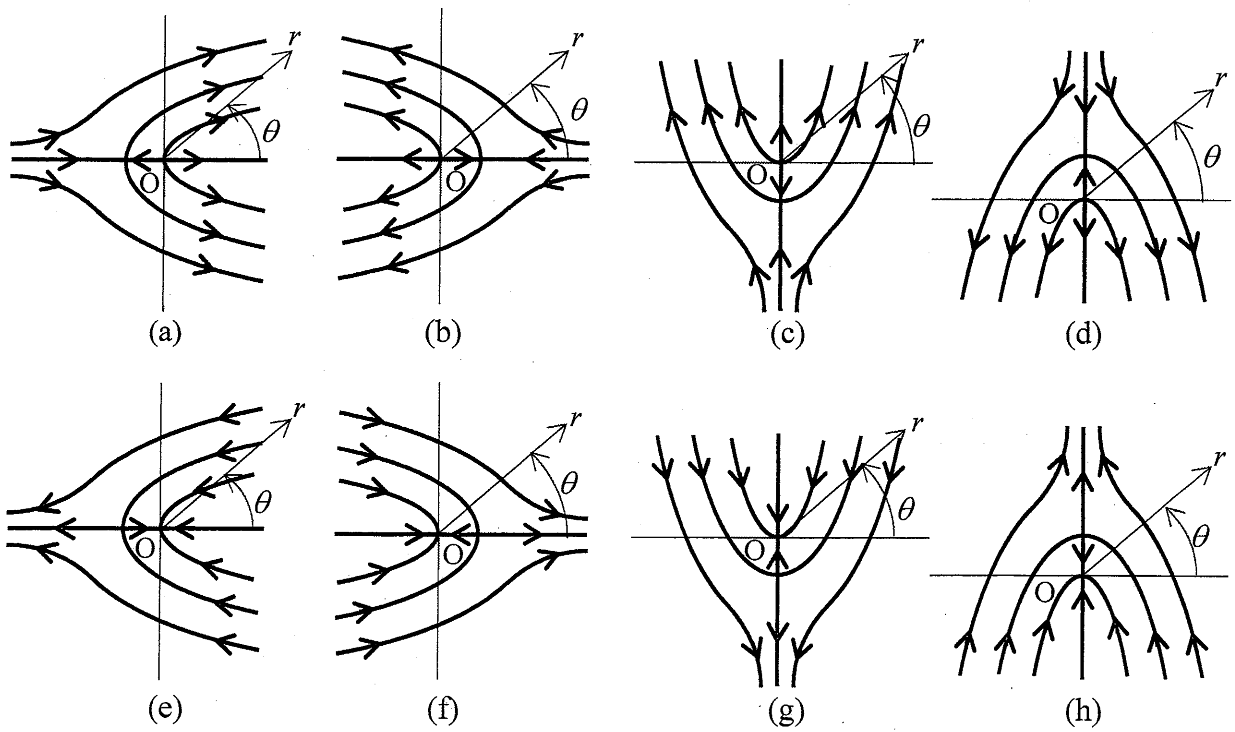


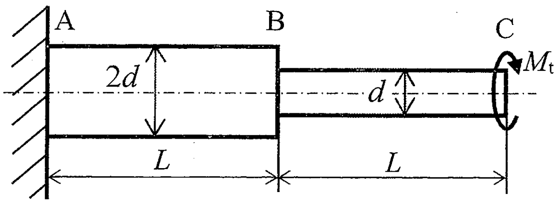
Fig. 2

材 料 力 学　STRENGTH OF MATERIALS

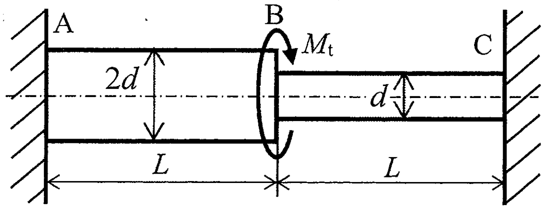
1. Consider a stepped solid circular shaft ABC as shown in Fig. 1. The length and diameter of portion AB are L and $2d$, and those of portion BC are L and d, respectively. The shear modulus of the shaft is G. Answer the following questions.

(1) As shown in Fig. 1(a), the stepped solid circular shaft ABC is fixed to a rigid wall at left end A, and is subjected to a twisting moment M_t at right end C. Determine the specific angles of twist of portion AB and BC, and the angle of twist at right end C.

(2) As shown in Fig. 1(b), the stepped solid circular shaft ABC is fixed to rigid walls at left end A and right end C, and is subjected to a twisting moment M_t at position B. Determine the reactive twisting moments M_{tA} and M_{tC} at left end A and right end C, and the maximum shear stress and maximum tensile stress of the shaft.



(a)



(b)

Fig. 1

2. Consider an L-shaped frame ABC of uniform bending stiffness EI in contact with a stepped floor ADEF as shown in Fig. 2. The L-shaped frame ABC consists of a beam AB of length h_1 and cross-sectional area A, and a beam BC of length l and cross-sectional area A, joined at point B. The entire beam BC is subjected to an equally distributed load w. The stepped floor height is h_2. Neglect the weight of the beams. Answer the following questions.

(1) Draw the shear force diagram (SFD) and bending moment diagram (BMD) of beam AB and beam BC, taking points A and B as origins, respectively.

(2) Find the shrinkage δ_B at point B of beam AB and deflection angle θ_B of beam AB.

(3) Find the equally distributed load w when the right end C of beam BC comes into contact with floor EF.

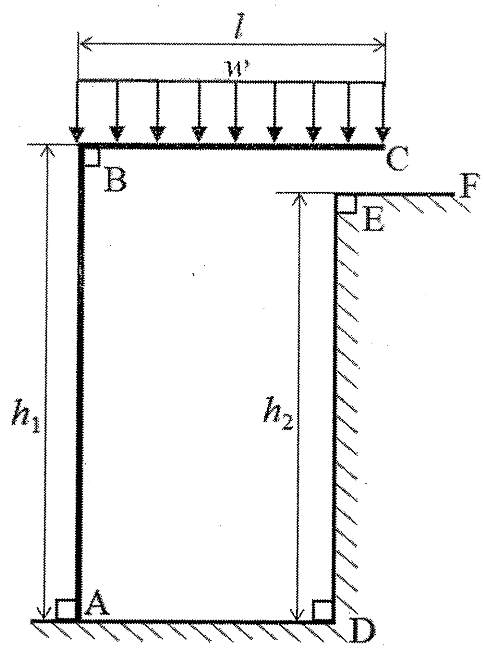


Fig. 2

機械力学 DYNAMICS OF MECHANICAL SYSTEMS

1. Consider a system consisting of a spring with spring constant k_1, a shaft with rotational stiffness k_2, a dashpot with damping coefficient c_1, a rotational dashpot with damping coefficient c_2, a rigid disk with radius r and mass moment of inertia J, and a mass m as shown in Fig. 1. A rope is wound around the disk rim and its end is tightly connected to the mass m. The masses of the spring, the shaft, the dashpots, and the rope are negligible. The rotational dashpot generates viscous damping moment represented as a product of angular velocity and damping coefficient c_2. The displacement of the mass m from the equilibrium position is denoted by x. Answer the following questions.

(1) Obtain the kinetic energy T of the system.

(2) Obtain the potential energy U of the system.

(3) Derive the equation of motion of the system.

(4) Obtain the conditions for the system to be overdamped, critically damped, and underdamped, respectively.

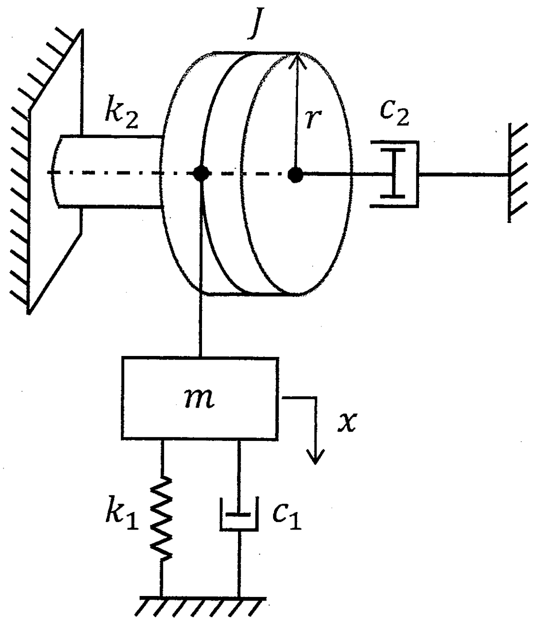


Fig. 1

2. Consider a system which consists of two trailers and two springs as shown in Fig. 2. Trailer 1 with mass m_1 is placed on a horizontal floor, and is connected to a wall by a spring with spring constant k_1. Trailer 2 with mass m_2 is placed on trailer 1, and connected to trailer 1 by a spring with spring constant k_2. The wall is oscillated horizontally by an external displacement $u = a \sin \omega t$, where a is the amplitude of the external displacement, ω is angular frequency, and t is time. The trailers move only in the horizontal direction with no friction. The absolute displacements of trailers 1 and 2 from the equilibrium positions are denoted by x_1 and x_2, respectively. The masses of the springs are negligible. When the system is in a steady state, answer the following questions.

(1) Derive the equation of motion for each trailer.

(2) Find the amplitudes of x_1 and x_2.

(3) Determine the angular frequency ω at which the amplitude of x_1 becomes zero. In addition, find the amplitude of x_2 at the angular frequency.

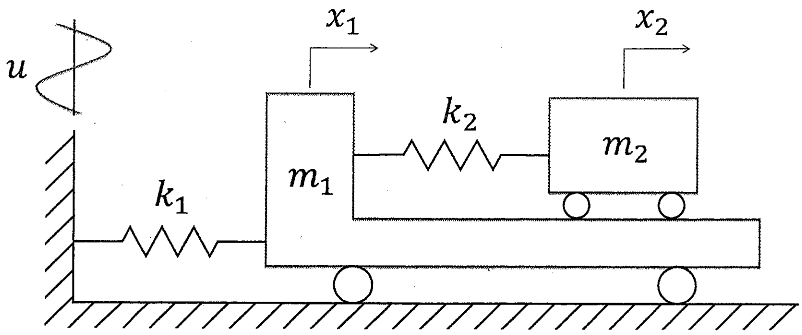


Fig. 2

制御工学 CONTROL ENGINEERING

1. Consider the closed-loop control system shown in Fig. 1. The open-loop transfer function $L(s)$ is given by

$$L(s) = \frac{K}{s(s+1)(s+4)},$$

where K is a positive constant. $R(s)$, $Y(s)$, and $E(s)$ denote the Laplace transforms of the reference input $r(t)$, the output $y(t)$, and the error $e(t)$, respectively. Solve the following problems.

(1) Derive the closed-loop transfer function $T(s)$ of this control system.

(2) Determine the range of values of K for which $T(s)$ is stable.

(3) Sketch the root locus of $T(s)$ as K in $L(s)$ varies. Also, find the value of K at the stability limit where the system exhibits sustained oscillations, and determine the corresponding angular frequency ω of the oscillation.

(4) When a unit ramp input $r(t) = t$ $(t \geq 0)$ is applied, calculate the steady-state value of the error $e(t)$.

(5) Suppose that the gain K in $L(s)$ is changed to a PI controller. Find $K(s)$ so that the closed-loop system is stable and the steady-state error to a unit ramp input becomes zero.

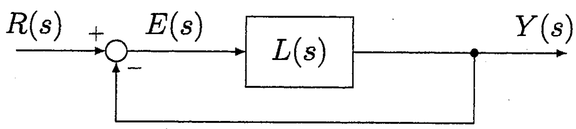


Fig. 1

2. Consider the system expressed by the state equations

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t), \quad y(t) = \boldsymbol{c}\boldsymbol{x}(t),$$

where $u(t)$ is the input, $y(t)$ is the output, $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)]^T$ is the state vector, and

$$\boldsymbol{A} = \begin{bmatrix} p & 1 \\ q & p \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ p \end{bmatrix}, \quad \text{and} \quad \boldsymbol{c} = \begin{bmatrix} p & 1 \end{bmatrix}.$$

Solve the following problems.

(1) Given $p \neq 0, q = -1$, find the transfer matrix $e^{\boldsymbol{A}t}$.

(2) Given $p = 2, q = -1$, find the transfer function from input to output.

(3) When $q = 1$ and given $\boldsymbol{A}$ and $\boldsymbol{b}$, there is a condition of p that arbitrary assignment of poles is not possible via state feedback. Find this condition.

(4) Consider the reason why the pole assignment cannot be achieved using state feedback for the condition derived in problem (3), where $p > 0$. Transform the system as $z_1 = x_1 + x_2$ and $z_2 = x_1 - x_2$, and answer the reason by drawing the state variable diagram to illustrate the relationship between the variables. As an example of a state variable diagram, the state variable diagram for $\dot{w} = 3w + u$ is shown in Fig. 2.

(5) Suppose a state feedback with $q = 1$,

$$u = -\boldsymbol{k}\boldsymbol{x}, \quad \boldsymbol{k} = \begin{bmatrix} k & -1 \end{bmatrix}$$

is applied to the system. The poles of the closed-loop system are assigned to -5 and -6. Find p and k with the condition of $p > 0$.

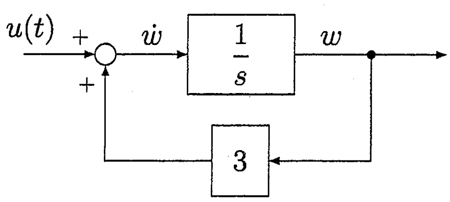


Fig. 2