

令和 8 年度(2026 年度)実施 秋季募集
東北大学大学院機械系 4 専攻入学試験

試験問題冊子

数学A MATHEMATICS A

2026 年 8 月 25 日(火)

Tuesday, August 25, 2026 9:30 - 10:30 (JST)

Notice

1. Do not open this test booklet until instructed to do so.
2. Put your examinee number on each of the answer sheets and the draft sheets.
3. Answer all the problems. Use two answer sheets for each problem.
4. At the end of the examination, double-check your examinee number and the problem numbers on the answer sheets. Put your answer sheets in numerical order on top of the draft sheets, place them beside the test booklet, and wait for collection. Do not leave your seat before instructed to do so.

数学 A MATHEMATICS A

1. Solve the following problems, where a and b are positive constants.

- (1) Show the Taylor series of the following function $f(x)$ about $x = 0$ up to the second order

$$f(x) = a^x.$$

- (2) Evaluate the following integral

$$\iint_D \log_e(e^{x^2} a^{y^2}) \, dx dy,$$

where the region D is given by

$$D = \{ (x, y) \mid x^2 + y^2 \leq 1, x \geq 0, y \geq 0 \}.$$

- (3) Evaluate the following limit

$$\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}}.$$

2. Two sequences $\{x_n\}$ and $\{y_n\}$ satisfy the following recurrence relations

$$\begin{cases} x_n = 4x_{n-1} + y_{n-1} \\ y_n = -x_{n-1} + 2y_{n-1} \end{cases} \quad (n = 1, 2, 3, \dots).$$

By using a vector $\mathbf{v}_n = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$, the relations can be rewritten as

$$\mathbf{v}_n = A\mathbf{v}_{n-1},$$

where A is a 2×2 matrix. Solve the following problems.

- (1) Find the eigenvalues of A .
- (2) Show that the Jordan normal form of A is $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$.
- (3) Obtain x_n and y_n when $x_0 = 2$ and $y_0 = 1$.

3. In the Cartesian coordinate system (x, y, z) , the vector field $\mathbf{A}$ is given by

$$\mathbf{A} = (x^2 - y^2) \mathbf{i} + z \mathbf{j} + (x^2 + y^2) \mathbf{k},$$

where $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the fundamental vectors in the x , y , and z directions, respectively.

The surface S is given by

$$S: z = x^3 - 3xy^2, \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1.$$

Solve the following problems.

- (1) Obtain $\nabla \cdot \mathbf{A}$ and $\nabla \times \mathbf{A}$.

- (2) Let C be the boundary of S on the plane $y = 0$. Evaluate the following line integral

$$\int_C \mathbf{A} \cdot d\mathbf{r},$$

where $\mathbf{r}$ is the position vector on C , and C is oriented such that x increases.

- (3) Evaluate the following surface integral

$$\int_S (\nabla \times \mathbf{A}) \cdot \mathbf{n} \, dS,$$

where $\mathbf{n}$ is the unit normal vector of S and the z component of $\mathbf{n}$ is positive.

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試験問題冊子

数学 B MATHEMATICS B

2026 年 8 月 25 日 (火)

Tuesday, August 25, 2026 13:00 - 14:00 (JST)

Notice

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数学 B MATHEMATICS B

1. Find the general solutions of the following ordinary differential equations.

(1) $\left(\frac{dy}{dx}\right)^2 - 3x^2 \frac{dy}{dx} + (\cos x) \frac{dy}{dx} = 3x^2 \cos x$

(2) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = \sin x$

2. Let the function $u(x, t)$ satisfy the partial differential equation

$$\frac{\partial u}{\partial t} + a \frac{\partial u}{\partial x} = \frac{\partial^2 u}{\partial x^2} \quad (at < x < 1 + at, \quad 0 < t),$$

where a is a positive constant. Considering the coordinate transformation $\xi = x - at$ and $\eta = t$, solve the following problems.

- (1) Show that the function $u(\xi, \eta)$ satisfies the following partial differential equation

$$\frac{\partial u}{\partial \eta} = \frac{\partial^2 u}{\partial \xi^2}.$$

- (2) When $u(\xi, \eta) = X(\xi)T(\eta)$, obtain the ordinary differential equations for $X(\xi)$ and $T(\eta)$, respectively.

- (3) Using the results of problem (2), find the general solution of $u(\xi, \eta)$.

- (4) Obtain $u(x, t)$ that satisfies the boundary conditions

$$u(at, t) = u(1 + at, t) = 0$$

and initial condition

$$u(x, 0) = \sin(\pi x).$$

3. The Laplace transform of a function $f(t)$ is defined by

$$\mathcal{L}[f(t)] = F(s) = \int_0^{\infty} f(t) e^{-st} dt.$$

The Heaviside function $u(t)$ is given by

$$u(t) = \begin{cases} 0 & (t < 0) \\ 1 & (t \geq 0). \end{cases}$$

Solve the following problems.

- (1) Derive the relationship

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s),$$

when the following equation holds true.

$$\lim_{s \rightarrow 0} \int_0^{\infty} \frac{df(t)}{dt} e^{-st} dt = \int_0^{\infty} \lim_{s \rightarrow 0} \left(\frac{df(t)}{dt} e^{-st} \right) dt.$$

- (2) Suppose that a function $x(t)$ satisfies the ordinary differential equation

$$\frac{d^2 x(t)}{dt^2} + 4 \frac{dx(t)}{dt} + kx(t) = u(t),$$

and the initial conditions

$$x(0) = 1 \quad \text{and} \quad \left. \frac{dx(t)}{dt} \right|_{t=0} = 0,$$

where k is a real number. Obtain the Laplace transform $X(s)$ of $x(t)$.

- (3) Obtain $x(t)$ when $k = 3$ in problem (2).

- (4) Obtain $\lim_{t \rightarrow \infty} x(t)$ when $k = 2$ in problem (2).